Published by Institute of Physics Publishing for SISSA/ISAS
Received: *March 12, 2004*
Accepted: *April 30, 2004*

Decoupling in an expanding universe: boundary RG-flow affects initial conditions for inflation

Koenraad Schalm
Institute for Strings, Cosmology and Astroparticle Physics, Department of Physics Columbia University, New York, NY 10027, U.S.A.
E-mail: kschalm@phys.colombia.edu

Gary Shiu
Department of Physics, University of Wisconsin Madison, WI 53706, U.S.A.
E-mail: shiu@physics.wisc.edu

Jan Pieter van der Schaar
Department of Physics, CERN, Theory Division 1211 Geneva 23, Switzerland
E-mail: jan.pieter.van.der.schaar@cern.ch

Abstract: We study decoupling in FRW spacetimes, emphasizing a lagrangian description throughout. To account for the vacuum choice ambiguity in cosmological settings, we introduce an arbitrary boundary action representing the initial conditions. RG flow in these spacetimes naturally affects the boundary interactions. As a consequence the boundary conditions are sensitive to high-energy physics through *irrelevant* terms in the boundary action. Using scalar field theory as an example, we derive the leading dimension four irrelevant boundary operators. We discuss how the known vacuum choices, e.g. the Bunch-Davies vacuum, appear in the lagrangian description and square with decoupling. For all choices of boundary conditions encoded by relevant boundary operators, of which the known ones are a subset, backreaction is under control. All, moreover, will *generically* feel the influence of high-energy physics through irrelevant (dimension four) boundary corrections. Having established a coherent effective field theory framework including the vacuum choice ambiguity, we derive an explicit expression for the power spectrum of inflationary density perturbations including the leading high energy corrections. In accordance with the dimensionality of the leading irrelevant operators, the effect of high energy physics is linearly proportional to the Hubble radius H and the scale of new physics $\ell = 1/M$.

Keywords: Renormalization Group, Cosmology of Theories beyond the SM.

http://jhep.sissa.it/archive/papers/jhep042004076/jhep042004076.pdf

JHEP04(2004)076

Contents

1. Introduction

String theory provides a fundamental framework to describe physics at the highest energy scales. Yet, the details of transplanckian physics have completely eluded us so far. Fortunately, the notion of decoupling allows us to understand low energy phenomena despite our ignorance of physics at very high energies. Renormalization Group (RG) flow teaches us that the effects of high energy physics can be captured by only a finite number of relevant couplings in the low energy theory. In flat spacetime, the decoupling between high and low energy physics is well established. However, for quantum field theories in curved space and in FRW universes in particular, decoupling is not so clearcut. In cosmological spacetimes high energy scales are redshifted to low energy scales via cosmic expansion. This connects high and low energy physics through unitary time evolution in addition to the dynamics. Decoupling, specifically in the inflationary context, is of great importance to upcoming cosmological precision experiments. All current physical scales would originate from transplanckian scales at the onset of inflation, if inflation lasted longer than the minimal number of e-folds. Conceivably, then, signatures of Planck scale physics (stringy or other) could show up in cosmological measurements [1]–[7]. This possibility whether glimpses of transplanckian physics can be observed in the cosmic microwave background (CMB) radiation [8] is determined by the strength with which transplanckian physics decouples. Remarkably, such effects *are* potentially observable, but only if the transplanckian physics selects a non-standard initial state [2, 6].[1] Other high energy effects are generically too small [4] (with the exception of the higher dimensional operators identified in [3]). More recently, explicit examples were presented to illustrate that the integrating out of a massive field could result in a non-trivial initial state, offering both a proof of principle that transplanckian physics may be observable, and suggesting that decoupling is more subtle in expanding universes [7].

In this article we would like to clarify the connections between vacuum/initial state selection and decoupling in a fixed FRW background (we ignore gravitational dynamics throughout). In cosmological settings, i.e. in a spatially homogeneous and isotropic universe, the size of the scale factor yields a preferred time coordinate, and as a consequence a hamiltonian approach has become standard [9]. In contrast to the hamiltonian point of view which emphasizes the dynamical evolution, a lagrangian point of view emphasizes the symmetries and scaling behaviour relevant to physical processes (see e.g. [4, 7, 10]). It is therefore the natural framework for a wilsonian RG understanding of decoupling of energy scales and relevant degrees of freedom determined by symmetries.[2] However, a lagrangian or an action by itself is insufficient to determine the full kinematic and dynamic behaviour of quantum fields. One must in addition specify the boundary conditions. This corresponds

[1]The nomenclature 'non-standard vacuum' state is also used. Strictly speaking there is no clear vacuum state in an FRW universe. In an abuse of language, we use vacuum and initial state interchangeably.

[2]Wilsonian RG in effect explains why (non-gravitational) physics works. Its success strongly suggests that the same principles are at work in quantum gravity and that general relativity is the low energy effective action relevant at scales below M_{Planck} (for a nice review on general relativity as an effective field theory see [11]). String theory, in particular, is an explicit manifestation of this idea.

to the choice of initial or vacuum state in the hamiltonian language. The question directly relevant to the window on transplanckian physics provided by inflation is therefore which *boundary conditions* to impose on the fields. To preserve the symmetries of the lagrangian a subset of all possible boundary conditions is often only allowed. With enough symmetry, e.g. Minkowski QFT, the choice may in fact be unique. FRW spacetimes have less symmetry and it is a priori not clear, what the natural or correct boundary conditions are. What we will explain in section 2 is that no matter which choice of boundary conditions is made in the full quantum theory, RG-flow in the effective low energy action will generically change these conditions. In particular high-energy physics will affect the boundary conditions through irrelevant corrections, which we derive. We apply these results in section 4 to the computation of the power spectrum of inflationary density perturbations. The leading irrelevant correction to the boundary conditions is of dimension four, and we therefore find that the power spectrum is subject to corrections of order H/M with M the scale of new physics. This is in accordance with earlier predictions that transplanckian effects are potentially observable [2, 6]. Importantly, we are able to derive this result purely within the framework of wilsonian effective field theory. This makes our answer predictive both in the sense that the parametric dependence of inflationary physics on high-energy is now manifest, and that the strength is computable in any theory where the high energy physics is explicitly known. Because our results are derived within the context of effective field theory, they provide a settlement to the debate [2, 4, 6, 12] whether H/M corrections are consistent with decoupling arguments. We conclude with an outlook where we will briefly comment on the relation of our results to consistency issues regarding (non-trivial) de Sitter invariant vacua known as α-states. We will, however, begin with a summary, lest the trees obscure the forest.

1.1 Summary of our results

Any boundary conditions one wishes to impose can be encoded in a boundary action. This is even true for the Minkowski vacuum (section 2.3). It has long been known that the couplings in such a boundary action are renormalized at the quantum level. Equivalently, a wilsonian approach to the effective action ought to result not only in a renormalization of the boundary couplings, but also in the generation of irrelevant boundary operators. Consider, for example, a two scalar field model with a mass separation $M_\chi \gg m_\phi$ and boundary and bulk interactions $S^{\text{int}} = -\int g\chi\phi - \oint \gamma\chi\phi$. This is exactly solvable, and upon integrating out χ, permitted when the cut-off scale $\Lambda \ll M_\chi$, one generates the boundary interactions

$$S_{\text{eff}} = \oint \frac{g\gamma}{M_\chi^2}\,\phi\frac{\Box^n}{M_\chi^{2n}}\phi\,. \tag{1.1}$$

We will describe and review the wilsonian effective action for theories with a boundary, including this example, in section 2.

The issue of (boundary) wilsonian decoupling is relevant to our understanding of cosmology. In an expanding universe, there is no unique vacuum state. In the lagrangian language, this translates to a lack of knowledge of the appropriate boundary conditions. Recall that *any* boundary conditions, including the 'Minkowski' ones, can be encoded in

JHEP04(2004)076

a boundary action. Wishing to emphasize the lagrangian viewpoint, where the study of decoupling is most natural, we add a boundary action with free parameters at a fixed but arbitrary time t_0.

Our limited understanding of high-energy physics in the very early universe can thus be accounted for by the inclusion of a boundary action in a cosmological effective lagrangian. Whichever boundary conditions we choose this boundary action to encode, they will be subject to renormalization. In particular, the details of the high-energy physics, which has been integrated out, will be encoded in irrelevant corrections to the boundary action. For $\mathbb{Z}_2$ symmetric scalar field theory the leading irrelevant boundary operators (that respect the homogeneity and isotropy of FRW cosmologies) are

$$S_{\text{bound}}^{\text{irr.op.}} = \oint d^3x \left[-\frac{\beta_{\|}}{2M}\partial^i\phi\partial_i\phi - \frac{\beta_\perp}{2M}\partial_n\phi\partial_n\phi - \frac{\beta_c}{2M}\phi\partial_n\partial_n\phi - \frac{\beta_4}{2M}\phi^4 \right], \tag{1.2}$$

where ∂_n is the normal derivative. These operators are of dimension four — one dimension higher than the boundary measure — and describe corrections of order $|\vec{k}|/M$ plus a boundary four-point interaction. For the momentum range of interest to the CMB, $|\vec{k}| \sim H$, where H is the Hubble parameter, the quadratic operators scale as H/M and they are therefore the primary candidates for witnessing consequences of high-energy physics in cosmological data. The leading bulk operator is of order H^2/M^2 and is generically beyond observational reach [4]. Computing the inflationary perturbation spectrum in a de Sitter background, including the corrections to Bunch-Davies boundary conditions due to the irrelevant operators (1.2), we find

$$P^{\text{dS}}_{\substack{\text{BD}+\\ \text{irr.op.}}} = P^{\text{dS}}_{\text{BD}} \left(1 - \frac{\pi}{4H} \left[\frac{\overline{H}^2_\nu(y_0)}{i} \left[\frac{\vec{k}_1^2(\beta_{\|}-\beta_c)}{a_0^2 M} + \frac{\kappa_{\text{BD}}^2\beta_\perp}{M} - \frac{\beta_c m^2}{M} - \kappa_{\text{BD}}\frac{3\beta_c H}{M} \right] + \text{c.c.} \right] \right), \tag{1.3}$$

with

$$\kappa_{\text{BD}} = \frac{d-1+2\nu}{2}H - \frac{|\vec{k}|}{a_0}\frac{\overline{H}_{\nu+1}(y_0)}{\overline{H}_\nu(y_0)}, \tag{1.4}$$

where $H_\nu(y_0)$ are Hankel functions at $y_0 = |\vec{k}|/a(\eta_0)H$ whose index $\nu(m^2)$ depends on the mass m^2. Crucial in our exposition will be the proof (section 2.2) that, despite appearances, this expression does not depend on the location of the boundary action y_0. Only the meaning of the initial conditions matters, not where they are imposed.

Eq. (1.3) is our main result. Having translated the cosmological vacuum choice ambiguity into an arbitrary boundary action, we conclude based on wilsonian decoupling that the leading irrelevant operators in FRW field theory are boundary operators at order H/M. Using optimistic but not untypical estimates of $H \sim 10^{14}$ GeV and $M \sim 10^{16}$ GeV (string scale), new (transplanckian) physics will *generically* affect the standard predictions of inflationary cosmology at the one-percent level. *Conversely, CMB observations with an accuracy of one percent or better can potentially measure effects of transplanckian physics.* Only for very special choices of initial conditions and transplanckian physics will this correction be absent.

We further identify the boundary conditions corresponding to several cosmological vacuum choices including the generalization of the "Minkowski-space" boundary conditions (sections 2.3 and 3.1). In the wilsonian effective lagrangian description it is clear that no vacuum is preselected by a consistency condition. Any boundary condition encoded by relevant operators is consistent, in the sense that the Minkowski space stress tensor counterterm generated with the appropriate boundary conditions will render the cosmological stress tensor finite as well (section 3). Backreaction is always under control. Which cosmological boundary conditions are the right ones to impose, requires just physical input, as it should be.

2. Decoupling in theories with a boundary: a review

The study of field theories is primarily concerned with Minkowski backgrounds, with the symmetry-compatible boundary conditions that the fields vanish at infinity.[3] Actions which contain explicit boundary interactions, however, have been studied in the past [13, 14, 15, 16], and are receiving renewed attention (see e.g. [17, 18, 19, 20, 21, 22]). One can use such boundary interactions to enforce whichever boundary conditions one wishes. Consider, for example, scalar $\lambda\phi^4$ theory on a semi-infinite space[4]

$$S_{\text{bulk}} = \int_{y_0 \leq y < \infty} d^3x dy \; -\frac{1}{2}(\partial_\mu\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 \,, \tag{2.1}$$

with the following boundary interactions added

$$S_{\text{boundary}} = \oint d^3x \; -\frac{\mu}{2}\phi\partial_n\phi - \frac{\kappa}{2}\phi^2 \,. \tag{2.2}$$

Here $\partial_n = \partial_y$ is the derivative normal to the boundary. Expanding the action to first order in $\phi + \delta\phi$, we find the usual equation of motion

$$\delta S_{\text{bulk}} = \int d^3x dy \, \delta\phi \left(\Box\phi - m^2\phi - \frac{\lambda}{3!}\phi^3 \right) , \tag{2.3}$$

plus the boundary conditions

$$\delta S_{\text{bound}} = \oint d^3x \; - \delta\phi \left(\frac{\mu+2}{2}\partial_n\phi + \kappa\phi \right) - \frac{\mu}{2}\phi\partial_n\delta\phi \,. \tag{2.4}$$

If we insist that the variations $\delta\phi$ are arbitrary and do not vanish on the boundary (which would correspond to imposing Dirichlet boundary conditions), it appears that μ must vanish for consistency. As we will see shortly, however, renormalization can produce counterterms proportional to μ and a more correct point of view is that ϕ can be discontinuously

[3] One may alternatively think of Minkowski space field theory as defined on a (infinite volume) torus ("putting it in a box"), which has no boundary at all.

[4] We choose lorentzian $+++-$ signature throughout the paper. Working with effective actions, we implicitly assume that all results can be obtained by a Wick rotation from euclidean space. Depending on whether the boundary under consideration is spacelike or timelike relative signs and factors of i will appear. Our focus will be on spacelike boundaries in particular since those have a natural interpretation as initial states in a hamiltonian description. We discuss details relating to the signature of time and the Wick rotation of timelike to spacelike boundaries in appendix B.

JHEP04(2004)076

redefined on the boundary [23], together with a redefinition of the couplings which absorbs μ:[5]

$$\begin{aligned}\phi(x,y) &\to \phi(x,y)+\alpha\theta(y_0-y)\phi(x,y_0)\,,\\ \kappa' &\equiv \kappa+\kappa\left(\alpha+\frac{\alpha^2}{4}\right)+\delta(0)\left(\frac{\alpha^2}{2}-\mu\alpha-\frac{\mu\alpha^2}{2}\right), \qquad \alpha=\frac{2\mu}{(2-\mu)}\,.\end{aligned} \tag{2.5}$$

This field redefinition can be interpreted as a shift of the boundary value of ϕ to the correct saddlepoint.[6] That this is the correct interpretation follows from the fact that we can also treat μ perturbatively as an interaction. A Feynman diagram computation will then yield an effective action with coupling κ'.[7] After this 'renormalization' the boundary term from partial integration is canonical

$$\delta S_{\rm bound}=\oint d^3x-\delta\phi\partial_n\phi-\kappa'\delta\phi\phi \tag{2.6}$$

which vanishes when

$$\partial_n\phi=-\kappa'\phi\,. \tag{2.7}$$

We see that the (renormalized) value of κ determines the boundary condition. For $\kappa=0$ we have Neumann boundary conditions, for $\kappa=\pm\infty$ the (particular) Dirichlet boundary condition $\phi(x,y_0)=0$, and for finite κ a mixture of the two. All possible (linear) boundary conditions are recovered. This is comforting as there are no other terms of order ϕ^2 compatible with the symmetries. In fact, the boundary action $S_{\rm bound}$ is the most general one we can write down, if we limit our attention to relevant operators[8] and require (for the sake of simplicity) that the action is also invariant under the bulk $\mathbb{Z}_2$ symmetry $\phi\leftrightarrow-\phi$. Of course, for a second order PDE one needs two boundary conditions. The other comes from the second boundary of integration. In the example above this is $y=\infty$. See appendix A for details.

RG arguments then tell us, that in a bounded space the terms in the boundary action, even if they were not present at the outset, would be generated as counterterms. They are necessary for the consistency of the theory. Let us show this explicitly. Suppose we start with Neumann boundary conditions: κ initially vanishes. By the method of images, the

[5] Here $\theta(y)$ is the step function, with $\theta(0)=1/2$ and $\partial_y\theta(y)=\delta(y)$. Recall that this distribution is of measure zero, i.e. $\int_{y_0}^{\infty}dy\theta(y_0-y)f(y)=0$. Of the bulk terms only the kinetic term is therefore affected by the shift. Also note that $\int_{y_0}^{\infty}\delta(y-y_0)f(y)=\frac{1}{2}f(y_0)$.

One can also find a redefinition of the type $\phi'(y)=\phi(y)+\alpha\theta(y_0-y)\phi(y)$, which is the correct one from the point of view of coarse graining and the distributional definitions for $\theta(y)$ and $\delta(y)$. Interestingly, the redefinitions required are the same.

[6] When counterterms of the form $\phi\partial_n\phi$ are required for renormalization, this shift of the background value for ϕ is thus a boundary analogue of the Coleman-Weinberg phenomenon.

[7] A perturbative comparison with Feynman diagrams, which we perform in appendix D, also explains the delta function at zero argument. It serves to make all distributions conform to the bare boundary condition $\partial_n\phi=-\kappa\phi$.

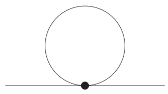
[8] We assume that the initial state encoded by the boundary action $S_{\rm bound}$ has no intrinsic size, i.e. a dimensionful scale. We are ultimately interested in vacuum-like initial conditions in cosmology. This restriction to scale-less initial states is therefore a natural one.

JHEP04(2004)076

Neumann propagator equals[9]

$$G_N(x_1,y_1;x_2,y_2) = -i\int \frac{d^3k_x dk_y}{(2\pi)^4}\frac{e^{ik_x(x_1-x_2)}\left(e^{ik_y(y_1-y_2)}+e^{ik_y(-y_1-y_2+2y_0)}\right)}{k_x^2+k_y^2+m^2}\,. \tag{2.8}$$

We will choose to regulate our theory by multiplying the propagator by a regulating function $\mathcal{F}(\Box/\Lambda^2)=\exp(-k^2/\Lambda^2)$ [24]. This makes the path integral well defined and cleanly separates out the ultraviolet divergences. The one-loop seagull graph then evaluates to



$$\begin{aligned} &= \langle\phi(x_1,y_1)\phi(x_2,y_2)\rangle_{\text{1-loop}} \\ &= \frac{-i\lambda}{4}G_N(x_1,y_1;x_1,y_1)\delta^3(x_1-x_2)\delta(y_1-y_2) \\ &= \frac{-\lambda\delta^3_{x;1,2}\delta_{y;1,2}}{4(2\pi)^4}\left[\int d^4k\frac{e^{-\frac{k^2}{\Lambda^2}}}{k^2+m^2}+\int d^3k_x dk_y\frac{e^{ik_y(-2y+2y_0)-\frac{k^2}{\Lambda^2}}}{k_x^2+k_y^2+m^2}\right]. \end{aligned} \tag{2.9}$$

The first term is the usual bulk $\lambda\phi^4$ divergence of the two-point function. The second term, however, is a newly divergent term, and quite obviously a direct consequence of the boundary conditions. Evaluating this term in more detail, we find

$$\begin{aligned} \langle\phi\phi\rangle_{\text{1-loop}} &= \frac{\lambda\delta^3_{x;1,2}\delta_{y;1,2}}{4(2\pi)^4}\left(\pi^{5/2}\Lambda e^{\frac{m^2}{\Lambda^2}}\right)\left(\frac{\Lambda}{\sqrt{\pi}}\int_0^1 ds e^{-s\Lambda^2(y_0-y)^2-\frac{m^2}{\Lambda^2 s}}\right) \\ &\sim \lambda\Lambda\delta^3_x\delta(y_1-y_2)\delta(y_1-y_0)\big|_{\Lambda^2\gg m^2}\,. \end{aligned} \tag{2.10}$$

Note that the new divergence is entirely located on the boundary. The last step utilizes one of the more common distributional definitions of the Dirac-delta function (before doing the finite integral over s). Recalling the coarse-graining steps underlying RG-flow, it should come as no surprise that the delta-function localization appears in a distributional limit. This simply reflects that our spatial resolution decreases under RG-flow, and the precise location of the boundary becomes fuzzy.

That the divergence is concentrated solely on the boundary (in this distributional sense) is reassuring. Bulk UV-physics should be unaffected by the presence of a boundary. It is precisely the breaking of Lorentz invariance due to the presence of the boundary that is responsible for the new divergence. By necessity it must then appear in the same sector of the theory that was responsible for the symmetry-violation in the first place.

To make the theory finite, we therefore need to add a boundary counterterm of the type[10]

$$S^{\text{count}}_{\text{bound}} = \oint_{y=y_0} d^3x\,\xi^2\left(\frac{m^2}{\Lambda^2}\right)\left(\frac{\lambda\Lambda}{\pi^{3/2}}\right)\phi^2\,. \tag{2.11}$$

with $\xi(m^2/\Lambda^2)$ chosen such that it cancels the divergence in eq. (2.10). This result is of course expected (in part) purely on dimensional grounds.

[9] Our domain of interest $y\in[y_0,\infty)$ is semi-infinite. Hence k_y is a continuous variable.

[10] Since the 'bare' boundary conditions are Neumann, this is the only type we can add.

The necessity of this counterterm has serious implications, however. Recalling the results from the first half of this section, we see that the boundary conditions *change* under RG-flow. In order to reproduce the same physics in a theory with a different cut-off, we not only need to change the vertices, but also the *boundary conditions*. (More precisely, to maintain a given physical renormalized boundary condition κ_{ren} we need to change the bare coupling κ.) Of course, this counterterm is scheme-dependent. The beta-functions at one loop on the other hand are scheme-independent, and we can extract the generic behaviour of the boundary conditions from them. We find that as we change the scale, the boundary conditions change under RG-flow as

$$\beta_\kappa \equiv \Lambda \frac{\partial \kappa}{\partial \Lambda}\bigg|_{m^2/\Lambda^2 \text{fixed}} = \xi^2 \Lambda \frac{\lambda}{\pi^{3/2}} + \mathcal{O}(\lambda^2)\,. \tag{2.12}$$

with $\xi^2 > 0$. This may seem surprising, but it does not go against the lore that boundary conditions are determined by physical conditions, and not by dynamics. It is worthwhile to repeat that what the RG-scaling of the boundary conditions says, is that in a *cut-off* theory, under a change of the cut-off, one reproduces the same physics when one changes the boundary conditions according to eq. (2.12).

2.1 Boundary RG fixed points and 'vacua'

A natural question to ask is what the endpoints of boundary RG-flow are. The explicit dimensionality of the coupling κ already betrays the answer. In the deep IR, when $|p| \ll \Lambda$ ($\Lambda \to \infty$ effectively; $m = \mu\Lambda$), κ blows up, and the boundary conditions tend to the special Dirichlet boundary condition $\phi(x, y_0) = 0$. Physically this is easily understood in wilsonian RG language. The moment the cut-off restricts the momentum scales $|p|$ to be smaller than m ($\Lambda \sim m$), all modes freeze out and the theory ceases to be dynamical. Hence the field ϕ 'vanishes', and must be Dirichlet.

Dirichlet conditions thus form a trivial fixed point of RG-flow. This is easily visible. When ϕ strictly vanishes on the boundary, simply no counterterms are possible. Both terms $\oint \phi\partial_n\phi$ and $\oint \phi^2$ vanish. For completeness, were one to repeat the computation eq. (2.9) for Dirichlet conditions, the difference is that the propagator now has a relative minus sign. As a consequence, the bulk divergence cancels the boundary divergence at $y = y_0$. Eq. (2.9) shows this clearly. In effective field theory the distinction between the fuzzy boundary and the bulk disappears in the deep IR limit, which explains why we can no longer treat bulk and boundary singularities separately when the boundary conditions become Dirichlet.

When the boundary is spacelike and represents initial conditions in time, the induced changes in the boundary conditions due to RG-flow have a natural description in the hamiltonian language of states. Under coarse graining the original state gets screened by vacuum polarization. In the low-energy effective theory, the correct state to use is a dressed version of the original state. If we take this picture further, we can deduce the boundary conditions which correspond to the vacuum. If the vacuum is the 'empty' state, then it ought not to become dressed under coarse graining. Translating back to the

JHEP04(2004)076

lagrangian language, this means that the corresponding boundary conditions will not suffer from renormalization. Hence a vacuum in the hamiltonian language should correspond to a fixed point of boundary RG-flow.[11]

2.2 Freedom of choice for the boundary location

What will be of fundamental importance to us, is that the location of the boundary is arbitrary. The introduction of a boundary action at y_0 is a way to encode the initial conditions at the level of the action, but it does not necessarily mean that there is a physical object or obstruction at $y = y_0$. It is simply a translation of the statement that a second order PDE needs two boundary conditions, but at what location one imposes those conditions is irrelevant. Of course, if one imposes the boundary conditions at a different location, they will not in general be of the same form as the original initial conditions. If one changes the location y_0 one must change the value of κ to keep the physics unchanged. A symmetry is therefore present between the location y_0 and κ.[12] To show this explicitly, choose a basis $\varphi_+(\vec{k}, y)$, $\varphi_-(\vec{k}, y) = \varphi_+^*(\vec{k}, y)$ for the two independent solutions of the kinetic operator. In terms of this basis, the linear combination which obeys the boundary condition $\partial_n\varphi(y_0) = -\kappa\varphi(y_0)$ is

$$\varphi_{b_\kappa}(\vec{k}, y) \equiv \varphi_+(\vec{k}, y) + b_\kappa(\vec{k})\varphi_-(\vec{k}, y)\,, \qquad b_\kappa(\vec{k}) = -\frac{\kappa\varphi_{+,0} + \partial_n\varphi_{+,0}}{\kappa\varphi_{-,0} + \partial_n\varphi_{-,0}}\,. \tag{2.13}$$

Here the subscript 0 means that the quantity is evaluated at the boundary y_0. Obviously if b_κ stays the same, physics stays the same. This allows us to derive a symmetry relation between the value κ and the location y_0. Under a constant shift of the boundary $\delta\varphi = \xi\partial_n\varphi = \xi\partial_y\varphi$ and a simultaneous change $\delta\kappa$, b_κ changes as[13]

$$\begin{aligned}\delta b_\kappa = -\xi&\left[\frac{\kappa\partial_n\varphi_{+,0} + \partial_n^2\varphi_{+,0}}{\kappa\varphi_{-,0} + \partial\varphi_{-,0}} - \frac{\kappa\varphi_{+,0} + \partial_n\varphi_{+,0}}{(\kappa\varphi_{-,0} + \partial_n\varphi_{-,0})^2}(\kappa\partial_n\varphi_{-,0} + \partial_n^2\varphi_{-,0})\right] - \\ -\delta\kappa&\left[\frac{\varphi_{+,0}}{\kappa\varphi_{-,0} + \partial_n\varphi_{-,0}} - \frac{\kappa\varphi_{+,0} + \partial_n\varphi_{+,0}}{(\kappa\varphi_{-,0} + \partial_n\varphi_{-,0})^2}(\varphi_{-,0})\right]\,.\end{aligned} \tag{2.14}$$

Demanding that δb_κ vanishes, one finds the change in κ necessary to keep physics unchanged under a change of the location of the boundary. This shows explicitly that this location is arbitrary.

2.3 Minkowski space boundary conditions

Minkowski space formally does not have a boundary of course. The arbitrariness of the location of the boundary, however, suggests that we should be able to treat it in a similar

[11] Presumably this is a UV-fixed point. Exciting the vacuum to a state, i.e. deforming away from the fixed point, reinstates RG-flow. The excitation, however, should not disappear in the deep IR. Hence the dressing of the state due to coarse graining leads one away from the vacuum. Of course to study boundary RG-flow, one needs an interacting theory. Any state in a free theory is a trivial fixed point of boundary RG-flow.

[12] This is not a true symmetry of the action. Because the coupling constant κ changes, it is an isomorphism between families of theories. This is analogous to general coordinate invariance of the target space manifold in non-linear sigma models.

[13] Note that b_κ depends on the basis choice $\varphi_\pm$, but κ does not.

JHEP04(2004)076

way. This is not quite manifest because, to stay within the framework of effective field theory, κ must remain an analytic dimension one operator in the spatial momenta. The symmetry (2.14) is subject to this condition. The harmonic oscillator boundary conditions, constructed here to yield physics equivalent to unbounded Minkowski space physics, will be consistent with this requirement. To find these conditions suppose the boundary is a fixed time slice. We can then take a cue from the hamiltonian formalism. Minkowski boundary conditions should correspond to choosing the standard Minkowski vacuum in the hamiltonian picture. By definition this is the state annihilated by the lowering operator of each spatial momentum mode $\vec{k}_x$ (in the free theory).

$$\hat{a}_{\vec{k}}|0\rangle = 0 \;\Leftrightarrow\; \left(\hat{\pi}_{\vec{k}} - i\omega(\vec{k},m)\hat{\phi}_{\vec{k}}\right)|0\rangle = 0\,, \qquad \omega(\vec{k},m) = \sqrt{\vec{k}^2 + m^2}\,. \tag{2.15}$$

The canonical momentum conjugate to $\pi_k = \partial_0\phi_k$ is precisely the normal derivative to the fixed time slice. This suggests that we should choose the spatial momentum dependent boundary conditions [25]

$$\partial_n\phi|_{y=y_0} = i\sqrt{\vec{k}^2+m^2}\,\phi|_{y=y_0} \;\longrightarrow\; \kappa = -i\sqrt{\vec{k}^2+m^2}\,. \tag{2.16}$$

This boundary condition descends from the 'higher derivative' operator $\oint \phi\sqrt{\partial_i^2 - m^2}\phi$. But, as κ has canonical dimension one, there is no new scale associated with this higher derivative term. Note that κ is purely imaginary. This is a consequence of imposing the boundary condition at a fixed time. Wick rotating from a spatial boundary with real κ generates a factor of i in the boundary condition $\partial\phi = -\kappa\phi$. We provide details behind this naive argument in appendix B. We show there that all correlation functions will be analytic in the boundary coupling κ, as is usual in effective field theory. We are therefore instructed to treat κ as real throughout all steps of the calculation, and only substitute its imaginary value at the end.

This momentum dependent choice of boundary conditions indeed ensures that the theory reproduces Minkowski space dynamics. For an arbitrary κ the Green's function is (see eq. (2.13), and recall that y parametrizes a timelike direction)

$$G_\kappa(x_1,y_1;x_2,y_2) = -i\int \frac{d^3\vec{k}dk_y}{(2\pi)^4}\frac{e^{i\vec{k}(x_1-x_2)}\left(e^{ik_y(y_1-y_2)} + \frac{ik_y+\kappa}{ik_y-\kappa}e^{ik(-y_1-y_2+2y_0)}\right)}{\vec{k}^2 - k_y^2 + m^2 - i\epsilon}\,, \tag{2.17}$$

where we have included the $i\epsilon$ term. The second term, at first sight, negates equivalence with the Minkowski propagator

$$G_{\rm Mink} = -i\int \frac{d^3\vec{k}dk_y}{(2\pi)^4}\frac{e^{i\vec{k}(x_1-x_2)+ik_y(y_1-y_2)}}{\vec{k}^2 - k_y^2 + m^2 - i\epsilon}\,, \tag{2.18}$$

The coefficient κ, however, is precisely chosen such that on shell the second term vanishes.[14] By unitarity, the theory with $\kappa = -i\omega(\vec{k},m)$ is then the same as the Minkowski space

[14] The second term only vanishes for the domain $\theta(y_1 + y_2 - 2y_0)$. Since our domain of interest is $y > y_0$, this is always true.

theory. We can see this explicitly by performing the integral over k_y. Doing so returns the standard Minkowski propagator in hamiltonian form

$$G(x_1, y_1; x_2, y_2) = \int \frac{d^3k}{(2\pi)^3} \frac{e^{i\vec{k}(\vec{x}_1-\vec{x}_2)-i\omega(\vec{k},m)(y_1-y_2)}}{2\omega} \theta(y_1 - y_2) + (y_2 \leftrightarrow y_1)\,, \tag{2.19}$$

which shows that the second term really is spurious. Indeed, this choice of κ removes the pole in the second term, which means its contribution to any physical quantity disappears.

We still have an official boundary at y_0 of course, even though the specific boundary conditions (2.16) ensure that it has no effect on physical amplitudes. The situation described here, is familiar from electrodynamics.[15] We have chosen an interface at y_0 where the dielectric properties happen to be the same for both materials. The transmission coefficient is therefore 100% and the wavefunction behaves as if the interface is not there, i.e. the interface is completely transparent.

2.3.1 Minkowski boundary conditions and RG-flow

Classical physics is indeed insensitive to a completely transparent interface. Is the quantum physics as well? In other words does the fact that the off-shell propagators appear to differ become relevant at the loop level? The answer is obviously no in perturbation theory. The cancellation of the pole by the specific 'Minkowski' choice for κ means that in any integral the contribution of the second term vanishes. Hence the Minkowski boundary conditions do not get renormalized. They are a fixed point of boundary RG-flow exactly as befits the boundary conditions corresponding to a true vacuum. The reason why this is so is clear. The choice $\kappa_{\text{Mink}} = -i\omega(\vec{k}, m)$ is precisely the one that restores the Lorentz symmetry naively broken by the introduction of a boundary. Counterterms are forbidden to appear for they would break the reinstated Lorentz symmetry.

2.4 Wilsonian RG-flow and irrelevant operators

Quite generically therefore the boundary conditions of a quantum field theory are affected by RG flow, unless they are protected by a symmetry. Integrating out high energy degrees of freedom necessitates a change in boundary conditions to reproduce the same physics in a low-energy effective description of the theory. Decoupling then ensures that the low-energy theory remains predictive: the effects of high-energy physics are primarily encoded in a small set of relevant operators with universal scaling behaviour independent of the details of the high-energy theory. Subleading corrections of an energy expansion are by definition captured by irrelevant operators. These encode the specifics of the high-energy completion of the theory.

One of our best hopes to detect the properties of high energy physics beyond the Planck scale is in a cosmological setting. The tremendous cosmological redshift during inflation may bring the consequences of such irrelevant operators within reach of experimental measurements. This exciting opportunity has been a preeminent question in recent literature. In section 4 we shall show that the irrelevant boundary operators discussed in

[15] Except that this boundary is spacelike, which is why we can in fact relate it to a choice of initial state.

JHEP04(2004)076

this subsection are responsible for the leading effects of high-energy physics in cosmology, appearing generically at order H/M_{Planck}. The leading irrelevant operators for the bulk theory have long been known and their consequences for cosmological measurements are discussed in [4]. However, it is well known that quantum field theory in cosmological settings suffers from a vacuum choice ambiguity. In the lagrangian language this corresponds to a choice of boundary conditions. As we have just seen, we can parametrize this ambiguity in the cosmological vacuum choice by adding an arbitrary boundary action $\oint \kappa\phi^2$. Whichever the value of κ may be, the influence of high-energy physics will be encoded in the irrelevant corrections to the boundary action. For that reason, we devote this section to a determination of the leading irrelevant operators on the boundary. Earlier studies have indeed indicated it is only (irrelevant) changes in the boundary condition which can have observable effects in measurements. Due to the symmetry constraints on the action the consequences of bulk irrelevant operators are just too small to be detectable. Our aim here is to provide a solid foundation for these earlier results.

One can make a straightforward guess as to what the leading boundary irrelevant operators are, insisting on locality, compatibility with the $\mathbb{Z}_2$ symmetry, and SO(3) rotational invariance on the boundary.[16] They are the dimension four operators:

$$\oint_{y=y_0} d^3x\, \phi^4\,, \qquad \oint_{y=y_0} d^3x\, \partial^i\phi\partial_i\phi\,, \qquad \oint_{y=y_0} d^3x\, \partial_n\phi\partial_n\phi\,, \qquad \oint_{y=y_0} d^3x\, \phi\partial_n\partial_n\phi\,. \tag{2.20}$$

Note that the breaking of Lorentz invariance on the boundary distinguishes normal and tangential derivatives, and that normal derivatives cannot be integrated by parts. Varying ϕ infinitesimally, the latter two will generate normal derivatives on the variation $\partial_n\delta\phi$. To restore the applicability of the calculus of variations, one needs to perform a discontinuous field redefinition and adjustment of the couplings similar to (2.5). (We do so in appendix C.) In this sense, all physics can be captured by the first two irrelevant operators. However, for tractability we will treat all four operators perturbatively and on the same footing. We will see in section 4 that these operators will lead to corrections of order H/M_{Planck} to inflationary density perturbations, as predicted by the studies [2]. Here we will give an explicit example where high-energy physics induces two of these dimension four irrelevant boundary operators.

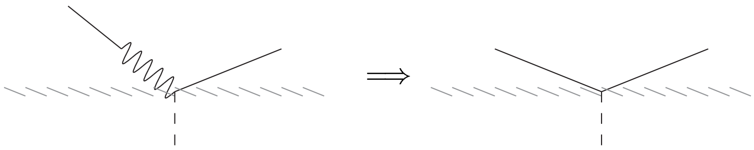
Tree-level diagrams exchanging a heavy field are the natural candidates for producing higher derivative corrections under RG-flow. We therefore add a scalar χ to the theory with mass $M_\chi \geq \Lambda$, to represent the high energy sector whose influence we will deduce. The only communication between the field χ and ϕ will be through the 'flavor-mixing' bulk and boundary couplings

$$S^{\text{int}}_{\text{high}} = -\int d^3x dy\, g\chi\phi - \oint d^3x\, \gamma\chi\phi\,, \tag{2.21}$$

and χ will have no other bulk or boundary (self)-interactions. Because the mass of χ is higher than the cut-off, it will not appear as a final state, and in this simple model we

[16] These symmetry constraints follow from the assumption that the initial state has no intrinsic dimensionful parameter. See footnote 8.

can integrate it out explicitly. Its influence on the low-energy effective $\lambda\phi^4$ theory is only through tree-level mass oscillation graphs and a boundary reflection. Treating the couplings g and γ as perturbations — hence the propagator for χ will have Neumann boundary conditions — consider the tree level correction to $\langle\phi\phi\rangle$ represented by the following Feynman diagram and its effective replacement.



(2.22)

Here wiggled lines denote the heavy field χ, solid lines the light field ϕ; the shaded region denotes the boundary, and the dashed line the insertion of a γ-vertex. This diagram is easily evaluated to

$$\begin{aligned}\langle\phi(x_1,y_1)\phi(x_2,y_2)\rangle_{\chi-effect} &= -2g\gamma G_N(x_1,y_1;x_2,y_0)\delta(y_2-y_0) \\ &= \frac{2ig\gamma\delta(y_2-y_0)}{(2\pi)^4}\left[\int d^4k\frac{e^{ik_x(x_1-x_2)+ik_y(y_1-y_0)-\frac{k^2}{\Lambda^2}}}{k^2+M_\chi^2}\right].\end{aligned} \tag{2.23}$$

Approximating the denominator in the standard way by a geometric series valid for $M_\chi^2 \gg \Lambda^2$,

$$\langle\phi\phi\rangle_\chi = \frac{2ig\gamma\delta_{y_2-y_0}}{M_\chi^2(2\pi)^4}\sum_{n=0}^{\infty}\left[\int d^4k\left(\frac{-k^2}{M_\chi^2}\right)^n e^{ik_x(x_1-x_2)+ik_y(y_1-y_0)-\frac{k^2}{\Lambda^2}}\right], \tag{2.24}$$

we extract the k_y dependence in the second term as a derivative to find[17]

$$\begin{aligned}\langle\phi\phi\rangle_\chi &= \frac{2ig\gamma\delta_{y_2-y_0}}{M_\chi^2(2\pi)^4}\sum_{n=0}^{\infty}\left[\left(\frac{\Box_1}{M_\chi^2}\right)^n\int d^4k e^{ik_x(x_1-x_2)+ik_y(y_1-y_0)-\frac{k^2}{\Lambda^2}}\right] \\ &= \frac{2ig\gamma\delta_{y_2-y_0}}{M_\chi^2(2\pi)^4}\sum_{n=0}^{\infty}\left[\left(\frac{\Box_1}{M_\chi^2}\right)^n\Lambda^4\pi^2 e^{-\Lambda^2\frac{(x_1-x_2)^2}{4}-\Lambda^2\frac{(y_1-y_0)^2}{4}}\right].\end{aligned} \tag{2.25}$$

Now recall that the projection onto the boundary of bulk terms appears as a distribution with resolution Λ. In this sense the above term contains the delta function $\frac{\Lambda}{2\sqrt{\pi}}e^{-\Lambda^2(y-y_0)^2/4}$. Up to this resolution the above expression is thus equivalent to

$$\langle\phi\phi\rangle_\chi = \frac{2ig\gamma\delta_{y_2-y_0}}{M_\chi^2}\sum_{n=0}^{\infty}\left[\left(\frac{\Box_1}{M_\chi^2}\right)^n\delta^3_\Lambda(x_1-x_2)\delta_\Lambda(y_1-y_0)\right]. \tag{2.26}$$

Hence we see explicitly the resultant higher derivative boundary interactions in the ϕ low-energy effective action. The above results correspond to the vertices

$$S_{\text{eff}} = \oint d^3x\frac{g\gamma}{M_\chi^4}\left[\partial_i\phi\partial^i\phi - \phi\partial_n\partial_n\phi\right] + \mathcal{O}\left(\left(\frac{\partial}{M}\right)^4\right). \tag{2.27}$$

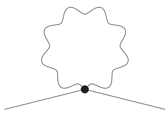
[17]Note that these results are not inconsistent with our earlier calculation (2.10). There we evaluate the answer in the approximation $\Lambda \gg m$. Here we approximate $\Lambda \ll M_\chi$. The exact intermediate answer obtained in eq. (2.10) is non-perturbative in Λ/M. This is why we approximate the momentum integral for $M_\chi \gg \Lambda$ in the standard way.

This supports the naive integrating out of χ after a shift $\chi \to \chi - g(\Box + M^2)^{-1}\phi$ as argued in section 1.1. The terms arising from the boundary term $\oint \gamma\chi\phi$ under this shift precisely reproduce the higher derivative terms (2.27).

Note the similarity between the expression (2.23) and the image-charge term in the seagull-graph (2.9). We see therefore that a similar set of higher derivative corrections can arise from *loop*-diagrams in a $\chi\phi$ theory with only the bulk interaction

$$S^{\text{int}}_{\text{high}} = \int d^3x dy - \widetilde{g}\chi^2\phi^2\,. \tag{2.28}$$

This is the hybrid inflation inspired model, considered before in the context of decoupling in FRW-spacetimes [7]. The seagull diagram responsible for the higher-derivative corrections is a direct copy of eq. (2.9) only to be evaluated in the limit $M_\chi \gg \Lambda$ rather than $m_\phi \ll \Lambda$.



$$\begin{aligned}
&= \langle\phi(x_1,y_1)\phi(x_2,y_2)\rangle_{\chi\text{-effect}} \\
&= -i\widetilde{g}G_N(x_1,y_1;x_1,y_1)\delta^3(x_1-x_2)\delta(y_1-y_2) \\
&= \frac{-\widetilde{g}\delta^3_{x;1,2}\delta_{y;1,2}}{(2\pi)^4}\left[\int d^4k\frac{e^{-\frac{k^2}{\Lambda^2}}}{k^2+M_\chi^2} + \int d^3k_x dk_y\frac{e^{ik_y(-2y+2y_0)-\frac{k^2}{\Lambda^2}}}{k_x^2+k_y^2+M_\chi^2}\right].
\end{aligned}\tag{2.29}$$

Repeating the geometric series expansion in k^2/M_χ^2,

$$\begin{aligned}
\langle\phi\phi\rangle_\chi = &\frac{-\widetilde{g}\delta^3_{x;1,2}\delta_{y;1,2}}{M_\chi^2(2\pi)^4}\times \\
&\times\sum_{n=0}^{\infty}\left[\int d^4k\left(\frac{-k^2}{M_\chi^2}\right)^n e^{-\frac{k^2}{\Lambda^2}} + \int d^3k_x dk_y\left(\frac{-k_x^2-k_y^2}{M_\chi^2}\right)^n e^{ik_y(-2y+2y_0)-\frac{k^2}{\Lambda^2}}\right].
\end{aligned}\tag{2.30}$$

we see that we can extract the k_y dependence in the second term as a derivative. The x dependence along the boundary and the full bulk term give purely local corrections as expected from loop graphs. Though this non-local y-dependence is counterintuitive, the physical reason is easily identified. It is the interaction with the image charge. We find

$$\begin{aligned}
\langle\phi\phi\rangle_\chi &= \text{bulk} + \frac{-\widetilde{g}\delta^3_{x;1,2}\delta_{y;1,2}}{M_\chi^2(2\pi)^4}\left[\sum_{n=0}^{\infty}\sum_{p=0}^{n}\binom{n}{p}\left(\frac{\partial_y^2}{M_\chi^2}\right)^p\int d^3k_x dk_y\left(\frac{-k_x^2}{M_\chi^2}\right)^{n-p} e^{ik_y(-2y+2y_0)-\frac{k^2}{\Lambda^2}}\right] \\
&= \text{bulk} + \frac{-\widetilde{g}\delta^3_{x;1,2}\delta_{y;1,2}\Lambda^3}{M_\chi^2(2\pi)^4}\left[\sum_{n=0}^{\infty}\sum_{p=0}^{n}\alpha_{n-p}\binom{n}{p}\left(\frac{\partial_y^2}{M_\chi^2}\right)^p\int dk_y e^{ik_y(-2y+2y_0)-\frac{k_y^2}{\Lambda^2}}\right] \\
&= \text{bulk} + \frac{-\widetilde{g}\delta^3_{x;1,2}\delta_{y;1,2}\Lambda^3\pi^{1/2}}{M_\chi^2(2\pi)^4}\left[\sum_{n=0}^{\infty}\sum_{p=0}^{n}\alpha_{n-p}\binom{n}{p}\left(\frac{\partial_y^2}{M_\chi^2}\right)^p\Lambda e^{-\Lambda^2(y-y_0)^2}\right].
\end{aligned}\tag{2.31}$$

where $\alpha_n = 2\pi^{3/2}(-2)^{n+1}(2n+1)!!$. In the distributional sense this is therefore equal to

$$\langle\phi\phi\rangle_\chi = \text{bulk} + \frac{-\widetilde{g}\Lambda^3}{M_\chi^2}\left[\sum_{p=0}^{\infty}\zeta_p\frac{\partial_y^{2p}}{M_\chi^{2p}}\delta(y-y_0)\right]. \tag{2.32}$$

where ζ_p can be read off from (2.31). The bulk one-loop χ-diagrams therefore gives rise to the higher-derivative irrelevant corrections on the boundary

$$S_{\text{eff}} = \sum_p \oint d^3x \frac{\widetilde{g}\beta_p\Lambda^3}{M_\chi^2}\phi\left(\frac{\partial_n^{2p}}{M_\chi^{2p}}\right)\phi\,. \tag{2.33}$$

This result shows that the boundary irrelevant operators will generically not appear in the combination $\oint \partial_i\phi\partial_i\phi - \phi\partial_n^2\phi$. This is a direct consequence of the fact that the boundary breaks Lorentz invariance. Examples which generate the other two irrelevant operators are easily found. The model just discussed will also generate $\oint \phi^4$ terms. A non-linear sigma model will naturally have $\oint \partial_n\phi\partial_n\phi$ corrections.

2.4.1 Minkowski space boundary conditions and irrelevant operators

An important question therefore is how generic the occurrence of irrelevant corrections is. In particular do fixed points of boundary RG-flow, e.g. the Minkowski boundary conditions or other 'vacua', still receive irrelevant corrections? RG principles tell us that we should expect them. Just because we are at a fixed point of RG-flow, does not mean that irrelevant operators encoding a high-energy sector are forbidden. In the context of boundary RG-flow, the connection between boundary conditions and 'vacua', makes this statement somewhat surprising. In Minkowski space in particular we do not expect that integrating out a high-energy sector would change the vacuum state in the low-energy effective theory even at the irrelevant level.[18] Both the general RG principles and the intuition that in Minkowski space high energy physics should not change the low-energy boundary conditions are true, as we will now illustrate. The first point is evident from the two scalar theory at the beginning of this section with the interactions given in (2.21). Integrating out the χ field exactly, clearly gives rise to the following irrelevant contributions to the low-energy effective theory for ϕ.

$$\begin{aligned} S^{\text{int}}_{\text{low-energy}} &= \frac{1}{2}\int d^3x dy - \phi(g+\gamma\delta(y-y_0))(\Box_{bc_\chi} - M_\chi^2)^{-1}(g+\gamma\delta(y-y_0))\phi \\ &= \text{bulk} + \sum_{n=0}^{\infty}\frac{1}{2}\oint \frac{2\gamma g}{M_\chi^2}\phi\left(\frac{\Box_{bc_\chi}}{M_\chi^2}\right)^n\phi + \frac{\gamma^2}{M_\chi^2}\phi\left(\frac{\Box_{bc_\chi}}{M_\chi^2}\right)^n\delta(0)\phi\,. \end{aligned} \tag{2.34}$$

Here $\Box_{bc_\chi}$ should be interpreted as acting on a complete set of eigenfunctions with the boundary conditions $\partial_n\chi = -\kappa\chi$ that belong to the massive field χ. To address the formal divergence of the delta function at its origin, $\delta(0)$, recall first that in a cut-off theory, as we are considering, all distributions become smeared on the scale of the cut-off. The $\delta(0)$ in the second term is therefore proportional to M_χ purely on dimensional grounds. Our cut-off scheme eq. (2.10) indicates that $\delta(x) = \lim_{\Lambda\to\infty}\pi^{-1/2}\Lambda e^{-\Lambda^2x^2}$, $\delta(0) = M\pi^{-1/2}$. This regularization only postpones the problem, however. In appendix D we perform a computation, which indicates that the $\delta(0)$ term arising from discontinuous field redefinitions does not explicitly appear in bulk correlation funcions. Its sole function is to generalize all distributions so that they obey the correct boundary conditions $\partial_n f(y) = -\kappa f(y)$.

[18] We thank Jim Cline for emphasizing this point.

JHEP04(2004)076

Consistent with the principles of decoupling, we see that whatever boundary conditions we choose for ϕ including fixed points of RG flow, the boundary action will receive irrelevant corrections. How can this possibly square with the idea that Minkowski space high energy physics should not correct the vacuum choice, i.e. the Minkowski space boundary conditions of ϕ? In this simple model it is fairly easy to see that the boundary conditions of ϕ change, because the massive field χ does not have Minkowski space boundary conditions. When χ is integrated out, this reverberates in the low energy effective boundary action for ϕ. A naive way to see that χ is not at a fixed point of boundary RG-flow, is to note that the full boundary condition for χ reads $\partial_n\chi = -\kappa\chi - \gamma\phi$. The explicit dependence on ϕ perturbs one away from a χ-sector fixed point κ_{fixed}. To consider a fixed point in the χ-sector alone is inconsistent of course; the full χ-ϕ dynamics needs to be taken into account. But an exact answer, possible because the theory is exactly solvable, shows that this naive guess is qualitatively correct. The exact answer is obtained by diagonalizing the theory to two fields Φ_1 and Φ_2 with action

$$\begin{aligned} S_{\text{bulk}} &= \frac{1}{2}\int d^3x dy\, \Phi_1\left(\Box - M_\chi^2 + \frac{g^2}{4M_\Delta^2}\right)\Phi_1 + \Phi_2\left(\Box - m_\phi^2 + \frac{g^2}{4M_\Delta^2}\right)\Phi_2 + \mathcal{O}(g^3)\,, \\ S_{\text{bound}} &= \frac{1}{2}\oint d^3x\, \Phi_1\left(\frac{2g\gamma}{M_\Delta^2} + \frac{\gamma^2\delta(0)}{M_\Delta^2}\right)\Phi_1 - \Phi_2\left(\frac{2g\gamma}{M_\Delta^2} + \frac{\gamma^2\delta(0)}{M_\Delta^2}\right)\Phi_2 + \mathcal{O}(\gamma^3, g\gamma^2, g^2\gamma)\,, \\ M_\Delta^2 &= M_\chi^2 - m_\phi^2\,. \end{aligned} \tag{2.35}$$

If we tune γ and g such that one of the two fields has Minkowski boundary conditions $\kappa_{\Phi_2} = -i\omega(\vec{k}, M_{\Phi_2})$, we see that the difference in masses $M_{\Phi_1} \sim M_\chi$ and $M_{\Phi_2} \sim m_\phi$ prevents the other from obeying Minkowski boundary conditions.

At a very fundamental level these results are easily understood. Recall that the Minkowski boundary conditions are the only boundary conditions respecting Lorentz invariance; this is what guarantees that the values of the boundary couplings correspond to a fixed point. The explicit boundary interaction $\oint -\gamma\chi\phi \simeq -\frac{1}{2}\int \delta(y-y_0)\gamma\chi\phi$ breaks Lorentz invariance, however. In the diagonal system with Φ_1, Φ_2, the Lorentz invariance is broken because one of the two fields does not obey Minkowski boundary conditions.

We have only shown that irrelevant operators will generically appear in a situation where a field in the high energy sector is not in the Minkowski vacuum. Lorentz symmetry should guarantee the converse: that if all massive fields obey Minkowski boundary conditions, no boundary RG-flow or boundary irrelevant operators can appear. Importantly, in the setting of interest to us, FRW cosmology, Lorentz invariance is absent. It is therefore not clear that cosmological boundary conditions, to which we turn now, are similarly protected from RG-flow and irrelevant contributions from high energy physics. Strictly applying the RG principles, we should *not* expect them to be protected.

3. Boundary conditions in cosmological effective lagrangians

We have seen that:

1. a boundary action can encode the boundary conditions one wishes to impose on the fields.

2. This holds in full generality. The boundary need not correspond to a physical obstruction or object. Completely transparent boundary conditions exist that mimick the situation as if there is no boundary. Introducing a boundary action to account for initial conditions therefore places no additional constraints on the theory.

3. Generically the boundary conditions will be affected by RG flow, and suffer irrelevant corrections that are controlled by the high energy physics.

We now use this knowledge to describe FRW cosmologies from a lagrangian point of view. The main issue in the hamiltonian description of FRW cosmologies is that of vacuum selection. In the absence of a global time-like Killing vector or asymptotic flatness, there is no unique vacuum state. There are two preferred candidates, the Bunch-Davies and the set of adiabatic vacuum states, which we review below, but some uncertainty remains. Whichever state is the true one, points (1) and (2) above tell us that we can account for this state by the introduction of a specific boundary condition at an arbitrary time t_0.

Our lack of knowledge of the specifics of the very early universe and the high energy degrees of freedom dominating at that time rather suggests to encode the initial state uncertainty in a 'past boundary' for any cosmological theory. With the boundary comes the lagrangian translation of the vacuum choice ambiguity: what boundary conditions to impose? We will not give an answer to this long-standing question. We will show, however, that whatever (local relevant) boundary conditions one chooses, they are consistent in the sense that the backreaction is under control. The counterterms *appropriate to the boundary conditions specified* that are necessary to render the Minkowski stress-tensor finite, do so in cosmological setting as well. This confirms the intuition that the boundary conditions do not affect UV-physics. And this continues to hold for any choice of cosmological initial conditions. This may come as a surprise. The Hadamard condition — that at short distances the two-point correlation function is the appropriate power of the geodesic distance $\sigma(x_1,x_2)^{d-2}$ — has long been thought to be a consistency requirement for cosmological boundary conditions. Only these correlation functions permit 'renormalization' by the standard Minkowski stress tensor. The lesson from section 2, however, is that other short distance behavior does not necessarily signal an inconsistency, but instead implies that the 'boundary conditions' need to be renormalized as well. This returns to the front the question which boundary conditions describe the physics of the real world, but *none* that can be deduced from local relevant boundary interactions are intrinsically inconsistent. This is the power of the effective lagrangian point of view.

Suppose for now that all choices for boundary conditions on the initial surface of an FRW universe are indeed consistent. Compared to Minkowski spacetime there is a new ingredient. The boundary condition needs to be covariantized. This is done by the introduction of a unit vector n^μ normal to the boundary.

$$\partial_n\phi \equiv n^\mu\partial_{\mu\phi} = 0\,, \qquad |g_{\mu\nu}n^\mu n^\nu| = 1\,. \tag{3.1}$$

In the conformal frame,

$$ds^2_{FRW} = a^2(\eta)(-d\eta^2 + dx^2_{d-1})\,, \tag{3.2}$$

JHEP04(2004)076

the unit normal vector to the boundary scales as a^{-1}. Hence the boundary condition reads

$$\frac{1}{a}\partial_\eta \phi|_{\eta=\eta_0} = -\kappa\phi|_{\eta=\eta_0}\,. \tag{3.3}$$

The explicit dependence on the scale factor a simply reflects that momenta redshift under cosmic expansion.[19] To construct the two-point correlation function for a massive scalar ϕ that satisfies this boundary condition, we need the equation of motion in an FRW background. For simplicity we will assume that this background is pure de Sitter; the results below generalize straightforwardly to power-law inflation and are therefore truly generic. The equation of motion is

$$\begin{aligned} &\frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu\phi(x,\eta) - m^2\phi(x,\eta) = 0\,, \\ \Rightarrow \qquad &\left(\frac{1}{a^2}\partial_\eta^2 + (d-2)\frac{a'}{a^3}\partial_\eta + \frac{\vec{k}^2}{a^2} + m^2\right)\phi(\vec{k},\eta) = 0\,. \end{aligned} \tag{3.4}$$

In the second step we Fourier transformed the spatial directions. Substituting the constant de Sitter Hubble radius $a^{-2}a' = H$, the explicit scale factor $a = -1/H\eta$ and making the conventional redefinition $\eta = -y/\vec{k}$, we have a Bessel equation for $\widetilde{\phi} \equiv y^{-(d-1)/2}\phi$:

$$\left(y^2\partial_y^2 + y\partial_y + y^2 + \frac{m^2}{H^2} - \frac{(d-1)^2}{4}\right)\widetilde{\phi}(\vec{k},y) = 0\,. \tag{3.5}$$

The most general solution to the field equation is therefore

$$\begin{aligned} \varphi_{b_\kappa}(\vec{k},\eta) &= \varphi_{\mathrm{dS},+} + b_\kappa\varphi_{\mathrm{dS},-} \\ \varphi_{\mathrm{dS},+} &\equiv (-\vec{k}\eta)^{(d-1)/2}\sqrt{\frac{\pi}{4\vec{k}}}\left(\frac{H}{\vec{k}}\right)^{\frac{d-2}{2}}\overline{H}_\nu(-\vec{k}\eta)\,, \qquad \nu = \sqrt{\frac{(d-1)^2}{4} - \frac{m^2}{H^2}}\,, \end{aligned} \tag{3.6}$$

with $H_\nu(y)$ the Hankel function satisfying eq. (3.5). The normalization and convention is such that in the limit $\vec{k} \to \infty$ we recover the Minkowski space solutions. The boundary conditions (3.3) determine b, as in eq. (2.13).

By construction the Green's function is given by (see appendix A for details)[20]

$$\begin{aligned} G_{\kappa_f,\kappa}(\vec{k}_1,\eta_1;\vec{k}_2,\eta_2) = (2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2)\mathcal{N}_{\kappa_f,\kappa}\Big(&\overline{\varphi}_{b_{\kappa_f}}(\vec{k}_1,\eta_1)\varphi_{b_\kappa}(\vec{k}_2,\eta_2)\theta(\eta_1-\eta_2) + \\ &+ \varphi_{b_\kappa}(\vec{k}_1,\eta_1)\overline{\varphi}_{b_{\kappa_f}}(\vec{k}_2,\eta_2)\theta(\eta_2-\eta_1)\Big)\,, \end{aligned} \tag{3.7}$$

[19] Realizing that cosmological scaling induces RG-flow we manifestly see the previous claim that Dirichlet conditions are trivial IR-fixed points.

[20] A 'covariant' Green's function is given by

$$G_{\kappa_f,\kappa}(\vec{k}_1,\eta_1;\vec{k}_2,\eta_2) = (2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2)\sum_n^{\mathrm{trunc}(\kappa_f)}\mu(n)\frac{\phi_{b_\kappa,n}(\eta_1)\phi_{b_\kappa,n}(\eta_2)}{H^2n^2 - m^2 + H^2(d-1)^2/4}\,.$$

where κ_f characterizes the future boundary condition at $\eta = \infty$ and $\mu(n)$ is an easily determined measure. From this expression it is clear that the delta function therefore also obeys the boundary condition. Indeed the delta function is best viewed as a completeness relation for eigenfunctions of the laplacian $\Box\varphi_k = -k^2\varphi$ obeying $a_0^{-1}\partial_\eta\varphi_k|_{\eta_0} = -\kappa\varphi_k|_{\eta_0}$, i.e.

$$\delta_\kappa(\eta_1-\eta_2) = \sum_n \mu(n)\phi_{b,n}(\eta_1)\overline{\varphi}_{b,n}(\eta_2)$$

where κ_f characterizes the future boundary conditions at $y = \infty$. The normalization $\mathcal{N}_{\kappa_f,\kappa}$ is chosen such that $(\Box - m^2)G = i\delta^d/\sqrt{-g}$. This requires that

$$\mathcal{N}_{\kappa_f,\kappa}\varphi_{b_\kappa}(\vec{k},\eta)\overset{\leftrightarrow}{\partial_\eta}\,\overline{\varphi}_{b_{\kappa_f}}(\vec{k},\eta) = -ia^{2-d}(\eta) = -i(-H\eta)^{d-2}\,. \tag{3.8}$$

We find that

$$\mathcal{N}_{\kappa_f,\kappa} = \frac{1}{(1-\overline{b}_{\kappa_f}b_\kappa)}\,. \tag{3.9}$$

From here on we will again restrict our attention to $d = 4$ spacetime dimensions.

3.1 Harmonic oscillator and shortest length boundary conditions

A special set of boundary conditions are the covariantization of the completely transparent "Minkowski" boundary conditions of eq. (2.15). We will call these "harmonic oscillator" boundary conditions. Recall that these correspond to the boundary action $\oint \phi\sqrt{\partial_i^2 - m^2}\phi$. Covariance requires that the scale factor should enter here as well. We thus find that the *cosmological* harmonic oscillator boundary condition is characterized by

$$\kappa_{HO} = -i\sqrt{\frac{\vec{k}^2}{a_0^2}+m^2}\,. \tag{3.10}$$

For the specific momentum dependent choice of boundary location $\eta_0^{SL}(\vec{k}) = -\Lambda/H|\vec{k}|$ or equivalently $a_0 = |\vec{k}|/\Lambda$, these boundary conditions correspond to a *constant* value for the physical parameter b. They are therefore the boundary conditions proposed in [2, 6]. Underlying this inspired choice is the thought that in a cosmological theory there is an 'earliest time', where a physical momentum $p \equiv \vec{k}/a(\eta)$ reaches the cut-off scale (the shortest length). Whether there is truly an earliest time in cosmological theories is an interesting question in its own right. It would be the natural location for the boundary action, but as a consequence of the symmetry between boundary location η_0 and coupling κ exposed in section 2.2, it is not directly relevant to us. Indeed it is easy to see that a momentum-independent coupling κ_{HO} at $\eta_0^{SL}(\vec{k}) = -\Lambda/H|\vec{k}|$ is equivalent to a boundary action on a standard time-slice η_0' with momentum-dependent coupling κ_{SL}

$$\kappa_{SL} = -\frac{\partial\phi_+(\eta_0') + b_{SL}\partial\phi_-(\eta_0')}{\phi_+(\eta_0') + b_{SL}\phi_-(\eta_0')}\,,\qquad b_{SL} = -\frac{\kappa_{HO}\phi_+(\eta_0^{SL}) + \partial\phi_+(\eta_0^{SL})}{\kappa_{HO}\phi_-(\eta_0^{SL}) + \partial\phi_-(\eta_0^{SL})}\,. \tag{3.11}$$

In the limit $\Lambda \to \infty$ we recover the harmonic oscillator vacuum at $\eta = -\infty$. The coupling κ' encodes these harmonic oscillator boundary conditions at $\eta_0 = -\infty$ in terms of conditions at η_0' *plus* corrections that vanish as $\Lambda \to \infty$. As we have seen in the previous section and will discuss in detail in the next, these corrections therefore correspond to the introduction of *specific irrelevant* boundary operators.

3.2 The Bunch-Davies and adiabatic boundary conditions

In universes without a global timelike Killing vector, there is no clear concept of the vacuum as a lowest energy state. Particle number is also not conserved and one cannot unambiguously define an 'empty' state either. Instead one must specify a particular in-state characterizing the initial conditions. Two solutions to this vacuum choice ambiguity have become preferred. One is the Bunch-Davies vacuum, which is indirectly constructed by requiring that for high momenta $\vec{k}/a \gg H$ the Green's function reduces to the Minkowski one. The second corresponds to the set of (n-th order) adiabatic vacua, which is constructed by the requirement that the number operator on the vacuum changes as slowly as possible [9, 26]. For de Sitter space the infinite order vacuum and the Bunch-Davies one are the same; we shall therefore only discuss the latter.

The boundary conditions corresponding to the Bunch-Davies vacuum are readily found. In the basis (3.6) we have chosen, the Bunch-Davies-state corresponds to choosing $b = 0$, and hence

$$\kappa_{\mathrm{BD}} = -\frac{\partial_n \varphi_{\mathrm{dS},+,0}}{\varphi_{\mathrm{dS},+,0}} . \tag{3.12}$$

Note that the Bunch-Davies boundary conditions are the analogues of the Minkowski boundary conditions in a mathematical sense only. The flat space Minkowski boundary conditions in eq. (2.16) are easily recognized as $\kappa_{\mathrm{Mink}}^{\text{flat-space}} = -\partial_n \varphi_{\mathrm{Mink},+,0}/\varphi_{\mathrm{Mink},+,0}$ with $\varphi_{\mathrm{Mink},\pm} \simeq e^{\pm i\omega t}$. Using the Bessel function recursion relation

$$\partial_y H_\nu(y) = \frac{\nu}{y} H_\nu(y) - H_{\nu+1} , \tag{3.13}$$

and the chain rule $\partial_\eta = -\vec{k}\partial_y$ (recall that $\partial_n = a^{-1}\partial_\eta$) a straightforward calculation yields

$$\begin{aligned} \kappa_{\mathrm{BD}} &= -\frac{\vec{k}}{a_0} \left(\frac{\overline{H}_{\nu+1}(-\vec{k}\eta_0)}{\overline{H}_\nu(-\vec{k}\eta_0)} + \frac{(d-1)+2\nu}{2\vec{k}\eta_0} \right) \\ &= -\frac{\vec{k}}{a_0} \left(\frac{\overline{H}_{\nu+1}(-\vec{k}\eta_0)}{\overline{H}_\nu(-\vec{k}\eta_0)} \right) + H\frac{(d-1)+2\nu}{2} . \end{aligned} \tag{3.14}$$

Knowing the asymptotes of the Hankel functions

$$z \to 0: \quad H_\nu(z) \sim -i\frac{1}{\sin(\nu\pi)\Gamma(1-\nu)} \left(\frac{2}{z}\right)^\nu = -i\frac{\Gamma(\nu)}{\pi} \left(\frac{2}{z}\right)^\nu , \tag{3.15}$$

$$z \to \infty: \quad H_\nu(z) \sim \sqrt{\frac{2}{\pi z}} e^{i(z - \frac{1}{2}\nu\pi - \frac{1}{4}\pi)} , \tag{3.16}$$

we see that for $\eta_0 \to -\infty$ the Bunch-Davies boundary condition reduces to harmonic oscillator boundary conditions

$$\begin{aligned} \kappa_{\mathrm{BD}} &\simeq -\frac{|\vec{k}|}{a_0} \left(e^{\frac{i\pi}{2}} \right) + H\frac{(d-1)+2\nu}{2} \\ &\simeq -i\frac{|\vec{k}|}{a_0} \end{aligned} \tag{3.17}$$

of a massless field. (One cannot say that the boundary conditions tend to Dirichlet, the diverging a_0 is compensated by the normal vector, see eq. (3.3).) The mass correction is subleading in this limit. We should keep in mind though that this is a formal expression. At $\eta_0 = -\infty$ the induced boundary volume vanishes, and boundary conditions cannot easily be accounted for in terms of a boundary action.

3.3 Transparent, thermal, adiabatic boundary conditions; fixed points of boundary RG flow?

The most natural choice for the boundary conditions are arguably the ones which are transparent. If there is no real interface at the boundary location y_0, no physical effects of its location should be noticeable. To define transparency we need a notion of incoming and outgoing waves. A clean definition of such waves only exists in asymptotically flat spaces. Suppose one establishes these and let us call the incoming wave (from the past) φ_- and the outgoing φ_+. The transparent boundary conditions are then those with $b_\kappa = 0$. Of course de Sitter space is not asymptotically flat, but based on the asymptotic behavior of the Bessel functions, one can argue that the basis functions $\varphi_{\mathrm{dS},-}$ and $\varphi_{\mathrm{dS},+}$ defined in (3.6) correspond to in- and out-going waves respectively. In that sense the Bunch-Davies boundary conditions are the transparent ones.

A definition which is more intrinsic to de Sitter is that the Bunch-Davies boundary conditions are the thermal boundary conditions. This emphasizes the existence of a cosmological horizon, and is probably tied to the notion of transparency. From the lagrangian point of view the true vacuum should be a (UV) fixed point of boundary RG-flow. In the presence of a global timelike Killing vector with a conserved quantum number $\partial_t\phi = iE\phi$ such a fixed point is easily constructed following the Minkowski space example in section 2.3. In cosmological spacetimes it is not clear what the fixed points of boundary RG-flow are or whether there are any. The absence of a unique vacuum suggests that there may be none. If we recall that cosmological expansion induces RG-flow, the definition of the adiabatic vacuum, i.e. that the number operator on the vacuum change as slowly as possible, becomes very interesting. It would be worthwhile to investigate these connections between the transparent (i.e. Bunch-Davies), the thermal, and the adiabatic vacuum in FRW backgrounds and fixed points of boundary RG-flow further.

3.4 Backreaction and renormalizability for arbitrary boundary conditions

We shall now make a crucial point. Any cosmological boundary condition κ, provided it is a dimension-one analytic function of the spatial momenta, is consistent in the sense that backreaction is under control. The divergences appearing in the stress tensor must, of course, be regulated by the flat space counterterms of the *same* theory. This includes the boundary counterterms for $\oint \kappa\phi^2$ and $\oint \mu\phi\partial_n\phi$. Our review in section 2 has made this clear. In a rather coarse fashion we can also see this directly from the FRW Green's function in the limit of high (spatial) momentum — in as far as this limit exists in a cut-off theory. Using the asymptotic values of the Hankel functions, the basis functions $\phi_{\pm,\mathrm{dS}}(\vec{k},\eta)$ tend to massless Minkowski ones (the mass is negligible in the high momentum limit)

$$\vec{k} \to \infty : \phi_{\pm,\mathrm{dS}}(\vec{k},\eta) \simeq \frac{1}{\sqrt{2\vec{k}}}\frac{e^{\pm i\vec{k}\eta}}{a} = \frac{\phi_{\pm,\mathrm{Mink}}(\vec{k},\eta)}{a}\,. \tag{3.18}$$

The coefficient b encoding the effective boundary conditions for high-momentum modes therefore does not vanish, but reads

$$b = -\frac{\kappa\phi_{+,\mathrm{Mink},0} + a_0^{-1}\partial_\eta\phi_{+,\mathrm{Mink},0} - H\phi_{+,\mathrm{Mink},0}}{\kappa\phi_{-,\mathrm{Mink},0} + a_0^{-1}\partial_\eta\phi_{-,\mathrm{Mink},0} - H\phi_{-,\mathrm{Mink},0}}$$

$$= -\frac{a_0\kappa + i|\vec{k}| + a_0 H}{a_0\kappa - i|\vec{k}| + a_0 H} e^{2i|\vec{k}|\eta_0} \,. \tag{3.19}$$

The last terms in the numerator and the denominator are negligible in this limit $|\vec{k}| \gg aH$. They are remnants of the fact that the background breaks Lorentz invariance. The coefficient b thus does not vanish in the high momentum limit. Because a non-zero b means that there will be divergences in the theory *aside* from the 'Minkowski'-space divergences, it appears that any choice of boundary conditions with $b \neq 0$ is in trouble. In section 2 we reviewed, however, that this is not so. The additional divergences are localized on the boundary surface where the boundary conditions are imposed, and can be reabsorbed in a redefinition of the boundary couplings. Any choice for b (descending from a boundary coupling κ that is dimension one and analytic in the spatial momenta) is consistent.

One is tempted to conclude that for any boundary condition imposed at $\eta_0 = -\infty$, the high spatial momentum limit of b vanishes. This is true in the sense that if we keep κ fixed our flat space intuition, that boundary effects vanish when the boundary is moved off to infinity, continues to hold. However, this goes against the principles behind the framework we advocate here. In the sense of the symmetry between boundary location and boundary coupling κ, as explained in section 2.2, it is only the specific combination b_κ which matters. At what location η_0 one imposes the boundary conditions κ is immaterial to the physics.

The conclusion is that the answer to the question "what boundary conditions should we impose on quantum fields in FRW backgrounds" requires physics input rather than internal consistency. The Bunch-Davies vacuum certainly seems the closest analogue of Minkowski boundary conditions, even though it is not the naive covariantization of them. The similarity suggests that the Bunch-Davies boundary conditions may correspond to a fixed point of boundary RG-flow. At the same time Lorentz symmetry is still broken. If they are renormalized, it would suggest that they are not special in any sense.

4. Transplanckian effects in Inflation

Inflationary cosmologies are the leading candidates to solve the horizon and flatness problems of the Standard Model of Cosmology. Consistency with the observed spectrum of temperature fluctuations in the Cosmic Microwave Background (CMB) provides an estimate of the Hubble parameter H during inflation. Depending on the model, H can be as high as 10^{14} GeV. With the string scale $M_{\text{string}} = 10^{16}$ GeV as the scale of new physics, this means that the suppression factor H/M of irrelevant operators could optimistically be at the one-percent level. This opens a window of opportunity to *experimentally witness* effects of Planck scale physics [1]. Besides its theoretical appeal, inflation is also the leading candidate for early universe cosmology on experimental grounds. The most precise cosmological measurements to date, the temperature fluctuations in the CMB, advocate inflation. The CMB measurements are therefore also the most promising arena where remnants of transplanckian physics could show up. In inflationary cosmologies the CMB temperature fluctuations originate in quantum fluctuations during the inflationary era. The issue of vacuum selection in cosmological settings thus has immediate consequences for CMB predictions. At the classical level the Bunch-Davies choice is, for reasons reviewed in the previous section, the preferred one; it is the closest analogue to the Minkowski boundary conditions. Previous investigations into effects of Planck scale physics suggest that the CMB fluctuation spectrum is affected at leading order in $H/M_{\rm Planck}$ and that this effect is precisely due to the choice of vacuum [2, 6]. Due to our ignorance of the details of Planck scale physics (i.e. our lack of understanding of string theory in time-dependent settings), decoupling in effective field theory is arguably the framework in which transplanckian corrections must ultimately be understood [4]. By the addition of an arbitrary boundary action encoding the boundary conditions, we have put the issue of vacuum selection on a consistent footing with the ideas of effective field theory. In this comprehensive formulation, we can deduce systematically what the effect of Planck scale physics is on boundary conditions (vacuum selection) and whether its effect on CMB predictions is indeed leading compared to bulk corrections.[21]

The Planck scale physics is encoded in irrelevant operators. The leading bulk irrelevant operator $\frac{1}{M^2}\int \phi\Box^2\phi$ consistent with the symmetries is dimension six. In section 2.4 we constructed and derived the four leading irrelevant boundary operators in flat space

$$\frac{1}{M}\oint_{y=y_0} d^3x\,\phi^4\,,\quad \frac{1}{M}\oint_{y=y_0} d^3x\,\partial^i\phi\partial_i\phi\,,\quad \frac{1}{M}\oint_{y=y_0} d^3x\,\partial_n\phi\partial_n\phi\,,\quad \frac{1}{M}\oint_{y=y_0} d^3x\,\phi\partial_n\partial_n\phi\,. \tag{4.1}$$

compatible with unbroken ISO(3) symmetry. In a cosmological setting this is the requirement of homogeneity and isotropy. These operators are all dimension four and as the explicit scaling shows, they are expected to be dominant over the leading bulk irrelevant operator. In curved space these operators are covariantized. For a scalar field ϕ covariantization has only a significant effect on the last operator in (4.1). A new coupling is needed which provides the connection for the covariant normal derivative

$$\frac{1}{M}\oint \sqrt{h}n^\mu n^\nu\left(\phi\partial_\mu\partial_\nu\phi-\phi\Gamma^\rho{}_{\mu\nu}\partial_\rho(g)\phi\right)=\frac{1}{M}\oint \sqrt{h}n^\mu n^\nu D_\mu\partial_\nu\phi\,. \tag{4.2}$$

Here $h_{ij}=g_{\mu\nu}\partial_i x^\mu\partial_j x^\nu$ is the induced metric on the boundary, and n^μ its unit normal vector. In FRW cosmology with the metric in the conformal gauge,

$$ds^2_{FRW}=a^2(\eta)(-d\eta^2+dx_3^2)\,, \tag{4.3}$$

and an initial timeslice $\eta=\eta_0$ as boundary, the induced metric, connection coefficients, and normal vector are

$$\begin{aligned} h_{ij} &= a_0^2(\delta_{ij})\,,\\ n^\mu &= a_0^{-1}\delta^\mu_\eta\,,\\ \Gamma^\eta{}_{ij} &= a_0H_0\delta_{ij}\,,\qquad \Gamma^i{}_{\eta j}=a_0H_0\delta^i_j\,,\qquad \Gamma^\eta{}_{\eta\eta}=a_0H_0\,. \end{aligned} \tag{4.4}$$

[21]The object of our study is an external scalar field in a fixed FRW background. Strictly speaking only the gravitational tensor fluctuations are effectively described by such a model. However, our arguments should apply to the scalar-metric fluctuations as well, since these only differ by an amplification factor of the inverse slow-roll parameter.

Here $a_0 \equiv a(\eta_0)$ and $H_0 = H(\eta_0)$ is the Hubble radius $H = a^{-2}\partial_\eta a$ at $\eta = \eta_0$. Substituting these values we obtain the FRW version of the irrelevant operator

$$\frac{1}{M}\oint a_0^3\phi\left(\partial_n - H\right)\partial_n\phi\,. \tag{4.5}$$

We shall compute the effect of the leading irrelevant operators on the two-point correlator of ϕ. In inflationary cosmologies, the latter determines the power spectrum of CMB density perturbations. We will assume we can treat the four-point bulk $\lambda\phi^4$ and (irrelevant) boundary interaction $\oint\phi^4$ perturbatively and will ignore them to first order. Combining the remaining irrelevant boundary operators in a correction to the FRW boundary action, one obtains

$$S^{\text{irr.op.}}_{\text{bound}} = \oint_{\eta=\eta_0} d^3x a_0\left[-\frac{\beta_\perp}{2M}\partial^i\phi\partial_i\phi - \frac{\beta_\|}{2M}\partial_\eta\phi\partial_\eta\phi - \frac{\beta_c}{2M}\phi D_\eta\partial_\eta\phi\right]. \tag{4.6}$$

The precise value of a coupling constants β_i is determined by *two* parts. (1) It is determined by the details of the transplanckian physics; e.g. if transplanckian physics is a free sector, decoupling is exact and $\beta = 0$ (for dynamical gravity the sectors are never decoupled of course), but (2) the couplings β_i are also covariant under the symmetry between boundary location and coupling. If we would have computed the irrelevant corrections to a boundary condition at a different location y_0', we would have found different values β_i which upheld that all physical quantities only depend on the choice of boundary location through a specific combination b_{κ,β_i}.

Two of the operators in eq. (4.6) contain normal derivatives. As discussed in section 2, such operators can be removed by a discontinuous field redefinition and a change of the remaining boundary couplings. We do so in appendix C. To lowest order in β_i/M, eq. (4.6) is equivalent to a boundary interaction (if the boundary coupling μ=0)

$$S^{\text{irr, leading}} = \oint a_0^3 d^3x\, -\frac{\phi^2}{2}\left[\frac{\vec{k}_1^2(\beta_\| - \beta_c)}{a_0^2 M} + \frac{\kappa^2\beta_\perp}{M} - \frac{\beta_c m^2}{M} - \kappa\frac{3\beta_c H}{M}\right], \tag{4.7}$$

where m^2 is the mass of the scalar field. Fourier transforming along the boundary, the leading irrelevant correction thus amounts to a change in the boundary condition κ by[22]

$$\kappa_{\text{eff}} = \kappa_0 + \frac{\vec{k}_1^2(\beta_\| - \beta_c)}{a_0^2 M} + \frac{\kappa_0^2\beta_\perp}{M} - \frac{\beta_c m^2}{M} - \kappa_0\frac{3\beta_c H}{M}\,. \tag{4.8}$$

We clearly see that the leading correction to the low-energy effective action occurs at order $|\vec{k}|/a_0 M$ and H/M. For CMB physics the momentum scale of interest is $|\vec{k}|/a_{\text{present}} \sim H$,

[22]Because the coupling κ is subject to renormalization, its value is fixed by a renormalization condition and an experimental measurement. An important question therefore is, whether the effects of irrelevant operators are experimentally measurable. The standard story, that (1) measured couplings always include all relevant and irrelevant corrections, and that (2) the contribution of each coupling β_i is an independent contribution to the precise running of coupling $\kappa_{\text{eff}}(\beta_i)$ under RG-flow, should apply. A very precise measurement of the scaling behaviour of κ should reveal the contributions of high energy physics encoded in the irrelevant operators.

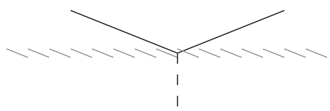
JHEP04(2004)076

and both are of the same order. The conclusion that the $|\vec{k}|$ dependent operators are suppressed by a factor a_0/a_{present} is incorrect, when we recall that the location of the boundary is arbitrary.

For a given FRW universe the Green's function, including the H/M correction to the boundary condition, can now simply be read off from eqs. (3.6)–(3.7). We can thus straightforwardly compute the leading transplanckian effect on the power spectrum of inflationary perturbations. The latter is related to the equal time Green's function with $\kappa_f = \bar{\kappa}$ (see appendix E)

$$\begin{aligned} P(\vec{k})_\kappa &= \lim_{\eta\to 0} \frac{\vec{k}^3}{2\pi^2} G_{\kappa_f=\bar{\kappa},\kappa}(\vec{k},\eta;-\vec{k},\eta) \\ &= \lim_{\eta\to 0} \frac{\vec{k}^3}{2\pi^2} \frac{|\varphi_{b_\kappa}(\vec{k},\eta)|^2}{(1-|b_\kappa|^2)}\,, \end{aligned} \tag{4.9}$$

where $\varphi_{b_\kappa}(\vec{k},\eta)$ is a solution to the (free) equation of motion, normalized according to the inner product (3.8), and with boundary condition $\partial_n\varphi| = -\kappa\varphi|$. *Note that the basis functions φ_{b_κ} only depend on the location of the boundary through the physical combination b_κ. This 'independence' of the location of the boundary guarantees that the power-spectrum — a physical quantity — is so as well.* For an infinitesimal change in the boundary condition κ, we can treat the vertex $\oint -\frac{1}{2}\delta\kappa\phi^2$ perturbatively, and the change in the power spectrum simply amounts to computing the following Feynman diagram.



(4.10)

This immediately illustrates that if $\delta\kappa$ is of order H/M, the change in the power spectrum will be of order H/M. For completeness, we compute the power spectrum by de Sitter Feynman diagrams in appendix E. With the effective change in κ corresponding to the contributions of the irrelevant operators β_i known, we can also simply expand the exact solution for the power spectrum for any κ. Choosing the Hankel functions as basis as in eq. (3.6), the solutions φ_{b_κ} are given by

$$\varphi_{b_\kappa} = \varphi_+ + b_\kappa\varphi_-\,, \quad b_\kappa = -\frac{\kappa\varphi_{+,0} + \partial_n\varphi_{+,0}}{\kappa\phi_{-,0} + \partial_n\phi_{-,0}}\,. \tag{4.11}$$

For an infinitesimal shift $\delta\kappa$ the power spectrum is thus

$$P(\vec{k})_{\kappa+\delta\kappa} = P(\vec{k})_\kappa + \lim_{\eta\to 0}\frac{\vec{k}^3}{2\pi^2}\left[\frac{\delta b}{(1-|b|^2)^2}\overline{\varphi}^2_{b_\kappa} + \text{c.c.}\right] + \mathcal{O}(\delta b^2)\,. \tag{4.12}$$

Substituting the de Sitter values computed in the previous section, and using that asymptotically (see (3.16))

$$\lim_{\eta\to 0}\overline{\varphi}_{b_\kappa,\mathrm{dS}} = \frac{(1-\bar{b})}{(b-1)}\lim_{\eta\to 0}\varphi_{b_\kappa,\mathrm{dS}}\,, \tag{4.13}$$

we find that

$$P(\vec{k})_{\kappa+\delta\kappa} = P_\kappa\left(1 + \frac{1}{(1-|b|^2)^2}\left[\delta b\frac{(1-\bar{b})}{(b-1)} + \text{c.c.}\right]\right)\,. \tag{4.14}$$

Recall from eq. (2.14) that

$$\delta b = -\frac{\delta\kappa\varphi_{+,0}}{\kappa\varphi_{-,0} + \partial_n\varphi_{-,0}} + \frac{\delta\kappa\varphi_{-,0}(\kappa\varphi_{+,0} + \partial\varphi_{+,0})}{(\kappa\varphi_{-,0} + \partial_n\varphi_{-,0})^2} \,. \tag{4.15}$$

We see explicitly that the change in the power spectrum is also linear in H/M.

For the preferred Bunch-Davies vacuum choice, where $b = 0$, the corrections thus become

$$P_{\mathrm{BD}+\delta\kappa}(\vec{k}) = P_{\mathrm{BD}}\left(1 + \left[\delta\kappa\frac{\varphi_{+,0}^2}{-\phi_{-,0}\partial_n\phi_{+,0} + \phi_{+,0}\partial_n\phi_{-,0}} + \mathrm{c.c.}\right]\right). \tag{4.16}$$

It appears we have introduced a dependence on the boundary location, but we should not forget that $\delta\kappa$ intrinsically depends on y_0 as well. The combination above is guaranteed to be independent of the boundary location. We recognize in the denominator the normalization condition (3.8) (with $\partial_n = a^{-1}\partial_\eta$). The expression therefore simplifies to

$$P_{\mathrm{BD}+\delta\kappa} = P_{\mathrm{BD}}\left(1 + \left[\delta\kappa\frac{\phi_{+,0}^2}{-ia_0^{-3}} + \mathrm{c.c.}\right] + \mathcal{O}(\delta\kappa^2)\right). \tag{4.17}$$

Restricting our attention to de Sitter space, we insert the explicit expressions for the basis functions ϕ_+ from eq. (3.6), and obtain, using that $a_0 = \vec{k}/Hy_0$,

$$P^{\mathrm{dS}}_{\mathrm{BD}+\delta\kappa} = P^{\mathrm{dS}}_{\mathrm{BD}}\left(1 - \left(\frac{\pi}{4H}\right)\left[\frac{\delta\kappa\overline{H}^2_\nu(y_0)}{i} + \mathrm{c.c}\right]\right). \tag{4.18}$$

Substituting the irrelevant operator induced $\delta\kappa$ from eq. (4.8), we compute the following corrections to the power spectrum

$$P^{\mathrm{dS}}_{\mathrm{BD}+\delta\kappa} = P^{\mathrm{dS}}_{\mathrm{BD}}\left(1 - \frac{\pi}{4H}\left[\frac{\overline{H}^2_\nu(y_0)}{i}\left[\frac{\vec{k}_1^2(\beta_{\|} - \beta_c)}{a_0^2 M} + \frac{\kappa_{\mathrm{BD}}^2\beta_\perp}{M} - \frac{\beta_c m^2}{M} - \kappa_{\mathrm{BD}}\frac{3\beta_c H}{M}\right] + \mathrm{c.c.}\right]\right), \tag{4.19}$$

with (eq. (3.14))

$$\kappa_{\mathrm{BD}} = \frac{d - 1 + 2\nu}{2}H - \frac{\vec{k}}{a_0}\frac{\overline{H}_{\nu+1}(y_0)}{\overline{H}_\nu(y_0)} \,. \tag{4.20}$$

This is our final result. Let us stress again, that the apparent dependence on the boundary location is only that. The boundary couplings β_i by construction compensate the y_0 dependence and the whole expression is independent of y_0.

5. Conclusion and outlook

The recent successes in CMB measurements exemplified by [8], have made the computation of inflationary density perturbations a focal point of research. The computation of these density perturbations suffers from a fundamental deficiency, however, that is at the same time a wondrous opportunity. The enormous cosmological redshifts push the energy levels beyond the bound of validity of general relativity, the framework in which these computations are done. From a field theoretic point of view general relativity can be viewed

JHEP04(2004)076

as the low energy effective action of a more fundamental consistent theory of quantum gravity. This effective action has higher order corrections which when re-included increase its range of validity. These higher order corrections encode the physics that is specific to quantum gravity. Hence understanding the way these higher order corrections affect the computation of inflationary density perturbations is both needed to restore consistency to the computation, and provides an opportunity to witness glimpses of Planck scale physics in a measurable quantity.

However, an action by itself is not sufficient to extract the physics of quantum fields. One must in addition specify a set of *boundary conditions*. Which boundary conditions to impose is always a physical question. In the hamiltonian language boundary conditions correspond to a choice of vacuum state. In cosmological settings, due to the lack of symmetries the correct choice of vacuum, i.e. boundary conditions, is ambiguous. A number of proposals, though, exist for the correct state. What we have discussed here, is that this vacuum choice ambiguity can be framed in terms of the arbitrariness of a boundary action. This puts the full physics in the form of a naturally coherent effective action. Deriving the power spectrum of inflationary density perturbations within this framework, the lowest order corrections are irrelevant boundary operators of order H/M_{Planck}. Because we are able to use the language of effective field theory, not only is the parametric dependence of the inflationary perturbation spectrum on high-energy physics known, the coefficients are also in principle computable from the high-energy sector that has been integrated out. RG-principles tell us that *generically* this coefficient will be non-zero, except for very special choices of initial conditions and high energy completions of the low energy theory. In cosmological spacetimes in particular the Lorentz symmetry which forbids the appearance of such corrections in flat Minkowski space is absent. This makes the prediction that we can potentially observe Planck scale physics in the cosmic sky quite strong, or equivalently the absence of these effects would constrain the possible high energy completions, i.e. string theory.

Several earlier investigations have shown that the effects related to a choice of initial conditions are not the only way in which high-energy physics can show up in cosmological measurements. Effects due to a non-vanishing classical expectation value of high- [7] or low-energy [3] fields, or a modified dispersion relation (see, e.g. [1]) can be of the same order. The former two should fit into our framework by the explicit introduction of sources. The latter presumes an all-order effective action, which is finite and therefore has a specific kinetic term $\mathcal{F}(\Box/\Lambda)$. The subleading effects in Λ obviously change the two-point correlation function and hence the power spectrum. In RG-terms a specific choice of regulator function $\mathcal{F}(\Box/\Lambda)$ corresponds to a specific choice of UV-completion of the theory. The relevant behaviour is universal and independent of the choice of $\mathcal{F}(\Box/\Lambda)$, but the irrelevant corrections are not, of course.

The introduction of a boundary action to account for the initial conditions, and its behaviour under RG-flow including irrelevant corrections begs for a comparison with the idea of holography. The latter suggests that (gravitational) theories in d-dimensional de Sitter space have a dual formulation as a (euclidean boundary) conformal field theory of dimension $d-1$ [27, 28]. The cosmological implications of this conjectured correspondence underline the universality and robustness of predictions for inflationary density perturbations precisely because they are related to RG characteristics in the dual $d-1$ dimensional theory [10, 29, 30]. These qualitative similarities are striking, but there are crucial differences with the approach put forth here. Holography interchanges the IR and UV properties of the dual theories. The UV physics of a three-dimensional euclidean field theory corresponds to the IR of the four-dimensional de Sitter gravity and vice versa. The holographic screen where the dual field theory lives corresponds to a boundary action in the de Sitter future. Its precise position defines the UV cut-off in the euclidean field theory that should completely describe the infinite interior (i.e. the past) of the de Sitter bulk gravity theory. Time evolution in the bulk is then interpreted as RG-flow in the boundary field theory, and so the IR physics in the field theory corresponds to the infinite past in the bulk. Instead the boundary actions considered in this paper are introduced only to encode the initial conditions in the past of the four dimensional de Sitter gravity theory. They are not dual descriptions of the bulk de Sitter theory, but are merely introduced as effective tools to describe the initial conditions in the bulk. Nevertheless, it would be very interesting to study how the results described in this paper should be interpreted from the point of view of a putative dual three-dimensional euclidean field theory.

That early times in cosmological theories are dominated by UV physics leads to a final open question. Do cut-off theories in a cosmological setting cease to be valid beyond an earliest time? Naively this is so, and that time would be a natural location for our boundary action. The freedom, however, to impose initial conditions where-ever one wishes, means that we do not need to answer this question to address the issue of boundary conditions in FRW universes. This fact is made manifest in the symmetry (2.14) between boundary location y_0 and boundary coupling κ. Physics depends only on the invariant combination $b_\kappa(y_0)$. With the effective field theory description in mind, and the idea that 'vacua' are boundary RG fixed points, a truly interesting question is whether such boundary conditions exist, and if so, how they are related to the known cosmological vacuum choices.

5.1 A comparison with previous results and the discussion on α-states

Much discussion has taken place in the recent literature on the consistency of so-called α-states in de Sitter space [4, 31]. Initial investigations into the sensitivity of inflationary perturbations to high energy physics found that in pure de Sitter the leading H/M corrections to the power spectrum can be interpreted as choosing the harmonic oscillator vacuum (section 3.1) at the naive earliest time $\eta_0(\vec{k}) = -\Lambda/H|\vec{k}|$ where the theory makes sense, rather than the Bunch-Davies choice [2, 6]. Imposing such boundary conditions in pure de Sitter can equivalently be interpreted as selecting a non-trivial de Sitter invariant vacuum state called an α-state [6]. Strictly speaking, the Shortest Length (SL) boundary conditions are only imposed on momentum modes below the cut-off scale Λ of the theory, and they are not true de Sitter α-states. Subject to this distinction, the purported inconsistency of α-states, particularly with respect to the decoupling of Planck scale physics [31], therefore would have major consequences. If α-states and other boundary conditions are all inconsistent, all high-energy physics would have to be encoded in bulk irrelevant operators. This would put transplanckian effects in the CMB perturbation spectrum beyond observational reach.

Let us put first, that our results form solid evidence for the presence of H/M effects affecting inflationary predictions for the CMB perturbation spectrum. As the explicit expression (4.19) we derive for the power spectrum shows, our results, though qualitatively similar, are quantitatively far more general from having 'chosen' an (cut-off) α-state. The coherent effective lagrangian approach followed here gives a precise answer which differs in general from the (earliest-time) α-state approach, but upholds the qualitative validity. One can certainly ask to what choice of 'vacuum state' our results correspond; given the physical parameter b_κ this is straigtforward to work out. The answer may be interesting from the point of view of hamiltonian dynamics, but as we have shown here, in the lagrangian language of boundary conditions, any initial state which can be described by a local relevant boundary coupling κ is consistent. *There is no need to know whether α-states are consistent to study transplanckian corrections to inflationary perturbations.*

At the same time, vacuum choices, α-states included, do correspond to boundary conditions.[23] And boundary conditions should not spoil decoupling, although there will be new effects, as we reviewed in section 2. Taking this lesson to heart, it is hard to see how (earliest-time) α-states could be inconsistent. A recent article [32] arguing for the consistency of α-vacua does not exactly follow the approach outlined here, but is very much in the spirit of introducing boundary counterterms. An answer, however, is provided by pursuing the discussion in section 3.1 further. The (cut-off) α-vacua correspond to choosing earliest-time boundary conditions in an effective theory below scale M with the physical parameter b_{SL} a constant number. The precise relation is that $b_{SL} = e^{\alpha}$. One then readily derives that an α-vacuum corresponds to a boundary coupling (see eq. (3.11))

$$\kappa_{SL} = -\frac{\partial_n \phi_+(\eta_0') + b_{SL}\partial_n \phi_-(\eta_0')}{\phi_+(\eta_0') + b_{SL}\phi_-(\eta_0')}\,. \tag{5.1}$$

Recall that b_{SL} is constant. To analyze the high spatial momentum behavior, we may therefore approximate the modefunctions $\phi_\pm(\eta_0')$ by their Minkowski counterparts. In this limit the boundary coupling κ_{SL} encoding α-states becomes

$$|\vec{k}| \to \infty, \qquad \kappa_{SL} \simeq -i\frac{|\vec{k}|}{a_0}\frac{e^{i|\vec{k}|\eta_0'} - b_{SL}e^{-i|\vec{k}|\eta_0'}}{e^{i|\vec{k}|\eta_0'} + b_{SL}e^{-i|\vec{k}|\eta_0'}}\,. \tag{5.2}$$

The boundary coupling κ_{SL} therefore has an infinite set of poles

$$|\vec{k}| = \frac{-1}{2\eta_0'}\left((2n+1)\pi + i\ln(b_{SL})\right), \qquad n \in \mathbb{Z}\,, \tag{5.3}$$

in the momentum plane. Clearly this boundary coupling corresponds to a non-local action. Cut-off α-states, i.e. shortest length boundary conditions, therefore fall outside the class of local relevant boundary conditions we study here. But are they inconsistent? Recall that the original studies [2, 6] argue that α-vacua should encode (first order) effects of high-energy physics in the spectrum of inflationary density perturbations. This point of

[23] We are grateful to Brian Greene both for emphasizing the importance in explicitly discussing the consistency of α-vacua and his help in resolving the issue.

view therefore states that by construction the boundary coupling κ_{SL} includes the effects of irrelevant *boundary operators*. We are therefore instructed to treat the non-local nature of the boundary coupling κ_{SL} in the low-energy effective action in the usual way. One expands around the origin $|\vec{k}| = 0$ in the momentum plane generating a series of higher derivative irrelevant boundary operators with specific leading coefficients β_i.[24] This expansion is valid as long as we limit the range of our effective action to the location of the first pole $|\vec{k}| = \frac{1}{2|\eta_0'|}\sqrt{|\pi + i\ln b_{SL}|^2}$, i.e. physical momenta are constrained to the range $|p_0| = |\frac{\vec{k}}{a_0}| \lesssim \frac{H}{2}|\ln b_{SL}|$. (Eq. (3.19) gives us $b_{SL} \simeq H/2Me^{-2iM/H - i\pi/2}$, and we recover the cut-off $|p| < M$.) The fact that the complicated pole structure of boundary couplings of alpha-vacua is highly specific (they ensure that (non-cut-off) α-vacua are invariant under de Sitter isometries) is not to the point in this perspective. It is then also clear why α-vacua are not renormalizable, in particular in the sense that the bare backreaction, the divergence in the stress tensor, is to leading order not identical to that in Minkowski space. Irrelevant operators correspond to non-renormalizable terms in the action. Because the pole structure of the boundary coupling κ reveals that α-states are correctly to be interpreted as encoding specific contributions from irrelevant operators, any correlation function computed with respect to the α-vacuum, includes the contribution from these irrelevant operators. It is therefore *expected* to be non-renormalizable. Obviously this does not mean that the α-vacua are inconsistent. As always in effective actions one must 'neglect' any contributions of irrelevant operators for the purposes of renormalization. They only make sense in a theory with a manifest cut-off [24]. Removing the cut-off, removes the irrelevant operators. Indeed the α-states proposed in [2, 6] with $b_{SL} \simeq H/2M$ are naturally in accordance with this precept.

In this sense, the (cut-off) α-vacua are therefore manifestly consistent in the framework put forth here. They simply correspond to a specific choice of leading and higher irrelevant boundary operators. Whatever they are is not very interesting from the perspective of effective field theory.[25] A specific choice for the irrelevant operators means having chosen a specific form for the high-energy transplanckian completion of the theory. But what this physics is, is precisely the knowledge we are after.

Acknowledgments

We thank Robert Brandenberger, Cliff Burgess, Chong-Sun Chu, Jim Cline, Richard Easther, Brian Greene and Erick Weinberg for comments. We are particularly grateful to

[24] It is not completely clear that this interpretation withstands close scrutiny. Most non-local terms in the effective action have real poles. Here we are confronted with imaginary poles. Perhaps α-vacua correspond to a high-energy completion with numerous unstable particles.

[25] They are $\beta_c = -\frac{i}{3}e^{-2iM/H}, \beta_{||} - \beta_{\perp} = \frac{7i}{3}e^{-2iM/H}$. Expanding around small $|b_{SL}| = H/2M$ and small $|\vec{k}| \ll Ha_0$, we see that

$$\kappa_{SL} = -i\frac{\vec{k}}{a_0}\left[1 - \frac{H}{iM}e^{-2iM/H} - \frac{2|\vec{k}|}{a_0 M}e^{-2iM/H}\right] = \kappa_{\rm BD} - \kappa_{\rm BD}\frac{H}{iM}e^{-2iM/H} - 2i\frac{\kappa_{\rm BD}^2}{M}e^{-2iM/H}\,.$$

Comparing with (4.8) we find the coefficients β_i.

the organizers and participants of the Amsterdam Summer Workshops on String Theory 2002 and 2003 and those of the String Cosmology Conference at Santa Barbara, October 2003. KS and JPvdS both thank the string theory group at the University of Wisconsin at Madison for hospitality. Likewise, G.S. thanks ISCAP and the theory group at Columbia University. K.S. acknowledges financial support from DOE grant DE-FG-02-92ER40699. The work of G.S. was supported in part by NSF CAREER Award No. PHY-0348093, a Research Innovation Award from Research Corporation and in part by funds from the University of Wisconsin.

A. Green's functions and boundary conditions

By definition the (real scalar) Green's function is the inverse of the (real scalar) kinetic operator[26]

$$(\Box_x - \omega^2)G(x,x') = i\frac{\delta^d(x-x')}{\sqrt{-g}} = (\Box_{x'} - \omega^2)G(x,x')\,, \qquad G(x,x') = G(x',x)\,. \quad \text{(A.1)}$$

For simplicity we reduce the spacetime to one timelike direction.

$$\left(\frac{d^2}{dt^2} + \omega^2\right) G(t,t') = -i\frac{\delta(t-t')}{\sqrt{-g(t)}}\,. \quad \text{(A.2)}$$

The Green's function is thus a solution to a inhomogeneous second order differential equation. The solution to (A.2) is therefore not unique; we can always add a combination of the two linearly independent homogeneous solutions (denoted by the superscript $(\mathfrak{h})$),

$$\left(\frac{d^2}{dt^2} + \omega^2\right) \phi_1^{(\mathfrak{h})}(t) = 0\,, \qquad \left(\frac{d^2}{dt^2} + \omega^2\right) \phi_2^{(\mathfrak{h})}(t) = 0\,, \qquad \phi_1^{(\mathfrak{h})}(t) \neq \phi_2^{(\mathfrak{h})}(t)\,, \quad \text{(A.3)}$$

to a Green's function and obtain another Green's function. This ambiguity is resolved by imposing a set of boundary conditions on the Green's function. Consistency then requires that the delta function appearing in eq. (A.2) obey these boundary conditions as well. Let the Green's function obey the boundary condition

$$\partial_t G(t,t')|_{t=t_0} = -\kappa_0 G(t_0,t')\,. \quad \text{(A.4)}$$

Acting with $\partial_{t'} + \kappa_0$ on $G(t,t')$ in eq. (A.2), which clearly commutes with $\frac{d^2}{dt^2} + \omega^2$, we are forced to conclude that

$$(\partial_{t'} + \kappa_0)\left(\frac{d^2}{dt^2} + \omega^2\right) G(t,t')|_{t'=t_0} = -i(\partial_{t'} + \kappa_0)\frac{\delta(t-t')}{\sqrt{-g(t)}}|_{t'=t_0} = 0\,. \quad \text{(A.5)}$$

The delta function on the r.h.s. of eq. (A.2) is therefore not the naive Dirac delta-function, but contains extra correction terms which contribute only on the boundary.

Limiting our attention to boundary conditions of the type eq. (A.4), i.e.

$$\begin{aligned}(\partial_t + \kappa_f)G_{\kappa_f,\kappa_0}(t,t')|_{t=t_f} &= 0\,,\\ (\partial_t + \kappa_0)G_{\kappa_f,\kappa_0}(t,t')|_{t=t_0} &= 0\,,\end{aligned} \quad \text{(A.6)}$$

[26] Recall that we are using the $++\,+-$ convention; i.e. in Minkowski space $\Box_x = -\partial_t^2 + \partial_{\vec{x}}^2$.

at the two boundaries at t_f and t_0, we solve the ambiguity in the Green's function in this appendix. We do so because this subset of possible boundary conditions is a very interesting one. It contains both the canonical Neumann and Dirichlet cases, and for $\mathbb{Z}_2$ symmetric scalars in effective field theory the boundary condition derived from all relevant boundary interactions are of this form.

There are two standard ways to solve for the Green's function. The hamiltonian way picks the timelike direction t as a the preferred one. In multiple dimensions the frequency ω is given by the eigenvalue of the remaining spatial component of the laplacian $-\omega^2\Phi_\omega(\vec{x},t) = \Box_{\vec{x}}\Phi_\omega(\vec{x},t)$. For the timelike direction, however, the hamiltonian Green's function uses as building blocks the two independent homogeneous solutions to the kinetic operator of eq. (A.3) multiplying stepfunctions $\theta(t-t')$ and $\theta(t'-t)$. With some insight the boundary conditions are readily imposed. Realize that for the past boundary condition only the part with $\theta(t'-t_0)$ will contribute; while for the future boundary condition only the part with $\theta(t_f-t')$ will contribute. Write the Green's function as

$$\begin{aligned} G^{\rm Ham}_{\kappa_f,\kappa_0}(t,t') &= \mathcal{N}^H_{\mathbb{R}}\left(\phi^{(\mathfrak{h})}_{b_f}(t)\phi^{(\mathfrak{h})}_{b_0}(t')\theta(t-t') + \phi^{(\mathfrak{h})}_{b_0}(t)\phi^{(\mathfrak{h})}_{b_f}(t')\theta(t'-t)\right) \\ &\equiv \mathcal{N}^H_{\mathbb{R}}\Big((\phi_1(t)+b_f\phi_2(t))(\phi_1(t')+b_0\phi_1(t'))\theta(t-t') + \\ &\qquad + (\phi_1(t)+b_0\phi_2(t))(\phi_1(t')+b_f\phi_2(t'))\theta(t'-t)\Big) \\ &= \mathcal{N}^H_{\mathbb{R}}\Big((b_0-b_f)\Big[\phi_1(t)\phi_2(t')\theta(t-t') + \phi_2(t)\phi_1(t')\theta(t'-t)\Big] + \\ &\qquad + (b_0-b_f)b_f\phi_2(t)\phi_2(t') + \phi_{b_f}(t)\phi_{b_f}(t')\Big)\,. \end{aligned} \tag{A.7}$$

Imposing the boundary conditions tells us that

$$\begin{aligned} \mathcal{N}^H_{\mathbb{R}}\phi^{(\mathfrak{h})}_{b_f}(t')\Big[(\partial_x+\kappa_0)(\phi^{(\mathfrak{h})}_1(t)+b^{(\mathfrak{h})}_0\phi^{(\mathfrak{h})}_2(t))_{t=t_0}\Big] &= 0\,, \\ \mathcal{N}^H_{\mathbb{R}}\phi^{(\mathfrak{h})}_{b_0}(t')\Big[(\partial_x+\kappa_f)(\phi^{(\mathfrak{h})}_1(t)+b^{(\mathfrak{h})}_f\phi^{(\mathfrak{h})}_2(t))_{t=t_f}\Big] &= 0\,. \end{aligned} \tag{A.8}$$

Hence if the linear combination of modes $\phi^{(\mathfrak{h})}_{b_0} \equiv \phi^{(\mathfrak{h})}_1 + b^{(\mathfrak{h})}_0\phi^{(\mathfrak{h})}_2$ satisfies the boundary condition $(\partial_t+\kappa_0)\phi^{(\mathfrak{h})}_{b_0}(t)|_{t=t_0} = 0$ at $t=t_0$, i.e.

$$b^{(\mathfrak{h})}_0 = -\frac{\kappa_0\phi^{(\mathfrak{h})}_1(t_0)+\partial_t\phi^{(\mathfrak{h})}_1(t_0)}{\kappa_0\phi^{(\mathfrak{h})}_2(t_0)+\partial_t\phi^{(\mathfrak{h})}_2(t_0)}\,, \tag{A.9}$$

and similarly for t_f, the boundary conditions are obeyed. Finally $\mathcal{N}^H_{\mathbb{R}}$ is determined by the normalization condition $(\frac{d^2}{dt^2}+\omega^2)G = -i\delta$.

$$\mathcal{N}^H_{\mathbb{R}} = \frac{i}{(b^{(\mathfrak{h})}_0 - b^{(\mathfrak{h})}_f)\left(\phi^{(\mathfrak{h})}_1\partial\phi^{(\mathfrak{h})}_2 - \phi^{(\mathfrak{h})}_2\partial\phi^{(\mathfrak{h})}_1\right)}\,. \tag{A.10}$$

It is a standard exercise to show that the Wronskian (the Klein-Gordon inner product) is independent of t.

The lagrangian way treats all directions on the same footing. Recall that the frequency ω is determined in terms of the eigenmodes of the spatial component of the laplacian.

The lagrangian Green's function similarly uses eigenfunctions of the temporal laplacian as building blocks,

$$\frac{d^2}{dt^2}\phi^{(n)}(t) = -\sigma^2(n)\phi^{(n)}(t)\,. \tag{A.11}$$

We assume that $\sigma(0) = 0$ for simplicity. For each n there will be two real solutions to the eigenfunction equation $\phi_1^{(n)}$ and $\phi_2^{(n)}$. Together they form an orthogonal and complete set over a covering space containing the domain $t \in [t_0, t_f]$ (in general with the measure $\sqrt{-g(t)}$)

$$\begin{aligned} \int_D dt\,\sqrt{-g}\phi_i^{(n)}(t)\phi_j^{(m)}(t) &= \frac{\delta_{m,n}\delta_{i,j}}{\mu(n)}\,, \\ \sum_{i,n}\mu(n)\phi_i^{(n)}(t)\phi_i^{(n)}(t') &= \frac{\delta(t-t')}{\sqrt{-g(t)}}\,. \end{aligned} \tag{A.12}$$

Here $\mu(n)$ is the measure on the 'dual' space. If the integral over t is over a non-compact domain, the sum over the eigenfunctions becomes an integral. Contrary to the statement below (A.3), the boundary conditions for the lagrangian Green's function are not satisfied by adding homogeneous terms. This can be traced back to the fact the delta function appearing in $(\frac{d^2}{dt^2}+\omega^2)G = -i\delta$ must obey the boundary conditions as well. This is done by the introduction of image charges in the covering space outside the domain $t \in [t_0, t_f]$ of interest. An immediate consequence of the fact that the covering space is larger than the domain $[t_0, t_f]$ is that one expects the mode sum will be truncated to only those modes which obey both boundary conditions. Here this is a direct consequence of the type of boundary conditions we are interested in. The conditions $(\partial_t + \kappa_{0,f})\phi|_{t=t_0,t_f} = 0$ clearly leave the normalization undetermined. For a linear combination which satisfies one boundary condition, only a subset of the modes will obey the other. Choosing modes which manifestly obey the boundary condition at $t = t_0$, this subset is of course those modes for which

$$\begin{aligned} &\partial_t\phi_1^{(n)}(t_f) + b_0^{(n)}\partial_t\phi_2^{(n)}(t_f) = -\kappa_f\left(\phi_1^{(n)}(t_f) + b_0^{(n)}\phi_2^{(n)}(t_f)\right) \\ \Rightarrow \qquad &b_0^{(n)} = -\frac{\kappa_f\phi_1^{(n)}(t_f) + \partial\phi_1^{(n)}(t_f)}{\kappa_f\phi_2^{(n)}(t_f) + \partial\phi_2^{(n)}(t_f)} \equiv b_f^{(n)}\,. \end{aligned} \tag{A.13}$$

The lagrangian Green's function then equals

$$G^{Lag}_{\kappa_f,\kappa_0}(t,t') = i\mathcal{N}^L_{\mathbb{R}} \sum_{n\neq\mathfrak{h}}^{\mathrm{trunc}(\kappa_f)} \mu(n)\frac{\phi_{b_0}^{(n)}(t)\phi_{b_0}^{(n)}(t')}{\sigma(n)^2-\omega^2}\,. \tag{A.14}$$

The normalization $\mathcal{N}^L_{\mathbb{R}}$ is determined by the condition $(\frac{d^2}{dt^2}+\omega^2)G = -i\delta$. For this one needs the explicit form of the eigenmodes.

Because the boundary conditions uniquely determine the solution to a second order PDE, the lagrangian Green's function eq. (A.14) and hamiltonian Green's function (A.7) are of course equal. In general this is difficult to show, but in specific cases one can do so. The most straightforward way is to decompose the hamiltonian Green's function into the complete set of modes $\phi_{b_0}^{(n_{\rm trunc})}$. If the domain x is non-compact and the mode sum in the lagrangian Green's function becomes an integral, contour integration is another way to show equivalence. This contour will reveal multiple step functions, which contribute on the boundary, but are always constant on the domain $[t_0, t_f]$. This way the lagrangian Green's function recovers the statement that homogeneous terms enforce the boundary conditions.

All these Green's functions can be written in a complex basis of eigenfunctions $\varphi_\pm = \phi_1 \pm i\phi_2$ (Note the script φ notation for complex eigenfunctions). Sometimes this is a more condensed notation. One easily computes that

$$\begin{aligned} \phi_{b_0} &\equiv \phi_1 + b_0^{\mathbb{R}}\phi_2\,, \qquad \text{with } b_0^{\mathbb{R}} = -\frac{\kappa_0\phi_1(t_0) + \partial\phi_1(t_0)}{\kappa_0\phi_2(t_0) + \partial\phi_2(t_0)} \\ &= \frac{1}{2}\Big((1 - ib_0^{\mathbb{R}})\varphi_+ + (1 + ib_0^{\mathbb{R}})\varphi_-\Big)\,. \end{aligned} \tag{A.15}$$

The combination $(1 \pm ib_0^{\mathbb{R}})$ can be related to the complex basis angle $b_0^{\mathbb{C}}$:

$$\frac{(1 + ib_0^{\mathbb{R}})}{(1 - ib_0^{\mathbb{R}})} = -\frac{\kappa_0\varphi_+(t_0) + \partial\varphi_+(t_0)}{\kappa_0\varphi_-(t_0) + \partial\varphi_-(t_0)} = b_0^{\mathbb{C}}\,. \tag{A.16}$$

Defining the complex analogue $\varphi_b = \varphi_+ + b^{\mathbb{C}}\varphi_-$, we find the relation between the real and complex bases,

$$\begin{aligned} \phi_{b_0} &= \mathcal{C}_{\kappa_0}(t_0)\varphi_{b_0} \\ &= \overline{\mathcal{C}}_{\kappa_0}(t_0)\overline{\varphi}_{b_0} = \mathcal{C}_{\kappa_0}(t_0)b_0^{\mathbb{C}}\overline{\varphi}_{b_0}\,, \\ \mathcal{C} &\equiv -\frac{(\kappa_0\varphi_-(t_0) + \partial\varphi_-(t_0))}{\kappa_0\varphi_+(t_0) + \partial\varphi_+(t_0) - (\kappa_0\varphi_-(t_0) + \partial\varphi_-(t_0))}\,. \end{aligned} \tag{A.17}$$

Note that for real κ the quantity $b_0^{\mathbb{C}}$ equals the inverse of its complex conjugate $b_0^{\mathbb{C}} = 1/\overline{b}^{\mathbb{C}}$. Hence $\overline{\varphi}_b \equiv \varphi_- + \overline{b}\varphi_+ = \frac{1}{b}(\varphi_+ + b\varphi_-) = \frac{1}{b}\varphi_b$. In appendix B we shall argue that κ should be treated as real throughout the calculation. For boundaries at a fixed time, i.e. initial conditions, κ will be imaginary, but the correct results are only reproduced if one analytically continues from real κ in the final correlation functions. We shall therefore treat κ as real always.

In this complex basis the hamiltonian Green's function equals

$$\begin{aligned} G^{Ham}_{\kappa_f,\kappa_0}(t,t') &= \mathcal{N}^H_{\mathbb{C}}\Big(\overline{\varphi}_{b_f}(t)\varphi_{b_i}(t')\theta(t-t') + \varphi_{b_i}(t)\overline{\varphi}_{b_f}(t')\theta(t'-t)\Big) \\ &= \frac{\mathcal{N}^H_{\mathbb{C}}}{b_f}\big(\varphi_{b_f}(t)\varphi_{b_i}(t')\theta(t-t') + \varphi_{b_i}(t)\varphi_{b_f}(t')\theta(t'-t)\big)\,. \end{aligned} \tag{A.18}$$

The normalization condition gives us that

$$\mathcal{N}^H_{\mathbb{C}} = \frac{-i}{(1 - b_i\overline{b}_f)\,(\varphi_+\partial\varphi_- - \varphi_-\partial\varphi_+)}\,. \tag{A.19}$$

The lagrangian Green's function in the complex basis is

$$G^{Lag}_{\kappa_f,\kappa_0}(t,t') = i\mathcal{N}^L_{\mathbb{C}}\sum_{n\neq\mathfrak{h}}\mu(n)\frac{\varphi^{(n)}_{b_0}(t)\overline{\varphi}^{(n)}_{b_0}(t')}{\sigma^2(n) - \omega^2}$$

$$= i\mathcal{N}^L_{\mathbb{C}} \sum_{n\neq\mathfrak{h}} \mu(n) \frac{1}{b_0^{(n)}} \frac{\varphi^{(n)}_{b_0}(t)\varphi^{(n)}_{b_0}(t')}{\sigma^2(n)-\omega^2}$$

$$= i\mathcal{N}^L_{\mathbb{C}} \sum_{n\neq\mathfrak{h}} \mu(n) \frac{\left(\varphi^{(n)}_+(t) + b_0^{(n)}\varphi^{(n)}_-(t)\right)\left(\varphi^{(n)}_-(t') + \bar{b}_0^{(n)}\varphi^{(n)}_+(t')\right)}{\sigma^2(n)-\omega^2}\,. \tag{A.20}$$

Again to determine $\mathcal{N}^L_{\mathbb{C}}$ one needs the explicit eigenfunctions: the terms with $\sum \varphi(t)_-\varphi(t')_-$, and $\sum\varphi(t)_+\varphi(t')_+$ generically do not vanish. If these terms solely contain contributions from the image charges, then $\mathcal{N}^L_{\mathbb{C}} \sim 1/(1+\bar{b}b)$.

A.1 Some simple examples

In flat space $\mathbb{R}^{d-1,1}$ the spatial and temporal components of the kinetic operator separate cleanly and the complex mode functions are $\varphi^{(\mathfrak{h})}_\pm = e^{\pm i\omega t}$; $\varphi^{(n)}_+(t) = e^{\mathrm{int}}$. Therefore $\varphi_+\partial\varphi_- - \varphi_-\partial\varphi_+ = -2i\omega$, $\int dt e^{i(n-m)t} = 2\pi\delta_{n,m}$, and $\sum_n \frac{1}{2\pi}e^{in(t-t')} = \delta(t-t')$. Furthermore

$$b^{(n)}_{0,f} = -\frac{\kappa_{0,f}+in}{\kappa_{0,f}-in}e^{2int_{0,f}}\,. \tag{A.21}$$

Thus the hamiltonian Green's function is

$$\begin{aligned} G^{\mathrm{Ham}}_{\kappa_f,\kappa_0}(t,t') = \frac{1}{2\omega(1-b_0\bar{b}_f)}\Bigg(\Bigg(&e^{-i\omega(t-t')} - \frac{\kappa_f - i\omega}{\kappa_f+i\omega}e^{-i\omega(2t_f-t-t')} - \\ &- \frac{\kappa_0+i\omega}{\kappa_0-i\omega}e^{-i\omega(t+t'-2t_0)} + \\ &+ \frac{\kappa_f-i\omega}{\kappa_f+i\omega}\frac{\kappa_0+i\omega}{\kappa_0-i\omega}e^{-i\omega(2t_f-2t_0-t+t')}\Bigg)\theta(t-t') + (t\leftrightarrow t')\Bigg). \end{aligned} \tag{A.22}$$

The lagrangian Green's function, on the other hand, gives

$$\begin{aligned} G^{\mathrm{Lag}}_{\kappa_f,\kappa_0}(t,t') &= i\mathcal{N}^L_{\mathbb{C}} \sum_{n\neq\omega}^{\mathrm{trunc},\kappa_f} \frac{1}{2\pi} \frac{e^{in(t-t')} - \frac{\kappa_0+in}{\kappa_0-in}e^{in(2t_0-t-t')} - \frac{\kappa_0-in}{\kappa_0+in}e^{in(t+t'-2t_0)} + e^{in(t'-t)}}{n^2-\omega^2} \\ &= i(2\mathcal{N}^L_{\mathbb{C}})\frac{1}{2\pi}\sum_{n\neq\omega}^{\mathrm{trunc},\kappa_f} \frac{e^{in(t-t')} - \frac{\kappa_0+in}{\kappa_0-in}e^{in(2t_0-t-t')}}{n^2-\omega^2}\,. \end{aligned} \tag{A.23}$$

Recall the admonition below eq. (A.17): κ is assumed to be real. We see that for flat space $\mathcal{N}^L_{\mathbb{C}} = 1/2$. The sum ranges over those modes for which

$$2i(n^2+\kappa_f\kappa_0)\sin(2n(t_f-t_0)) - 2in(\kappa_f-\kappa_0)\cos(2n(t_f-t_0)) = 0\,. \tag{A.24}$$

For $\kappa_f = 0 = \kappa_i$ (hence $b_{0,f} = e^{2\mathrm{int}_{0,f}}$), we indeed recognize the Green's function with Neumann boundary conditions at both ends:

$$\begin{aligned} G^{\mathrm{Ham}}_{\kappa_f,\kappa_0=0}(t,t') &= \frac{1}{2i\omega\sin(\omega(t_f-t_0))}\Big[\left(\cos(\omega(t_f-t_0-t+t'))\theta(t-t') + (t\leftrightarrow t')\right) + \\ &\qquad + \cos(\omega(t+t'-t_0-t_f)\Big]\,, \\ G^{\mathrm{Lag}}_{\kappa_f,\kappa_0=0} &= i\frac{1}{2\pi}\sum_{n\neq\omega}^{\mathrm{trunc},\kappa_f=0}\frac{e^{in(t-t')}+e^{in(2t_0-t-t')}}{n^2-\omega^2}\,, \end{aligned} \tag{A.25}$$

where the sum is over the modes $n = m\pi/2(t_f - t_0)$, $m \in \mathbb{Z}$. If we push the boundary t_f off to infinity, the mode sum becomes an integral. Evaluating this integral by contour integration we recognize the step functions in the hamiltonian Green's functions plus terms proportional to $\theta(2t_0 - t - t')$ and $\theta(t + t' - 2t_0)$, each multiplying homogenous solutions of the kinetic operator. For the domain of interest $t \in [t_0, \infty)$, only the term with $\theta(t+t'-2t_0)$ contributes, and we recover (part of) the homogeneous terms in the hamiltonian Green's function. Choosing $\kappa_f = \infty = \kappa_0$ (hence $b_{0,f} = -e^{2int_{0,f}}$) we similarly recover the doubly Dirichlet Green's function.

As we will show next (in appendix B) a particularly relevant choice of boundary couplings is $\kappa_0 = -\kappa_f = -i\omega$ (hence $b_0 = \bar{b}_f = 0$). From the expressions (A.22)-(A.23), we see that this choice reproduces the flat Minkowski space Green's functions (subject to the mode selection rule (A.24) reflecting the finiteness of the domain $[t_0, t_f]$). Note that we obtained this result without an $i\epsilon$ precription. This should be no surprise. The primary purpose of the $i\epsilon$ prescription is precisely to ensure that the Green's function obeys the right boundary conditions.

B. Initial states in transition amplitudes, path integrals and fixed time-slice boundaries

A naive Wick-rotation argues that the boundary action coupling constant κ is imaginary for boundaries in time. For a real scalar field such a boundary condition,

$$\partial_t \phi = -i|\kappa|\phi\,, \tag{B.1}$$

is at first sight inconsistent. We have stated that analyticity in the coupling constants for correlators computed in perturbation theory provides a resolution. One should treat κ as real, and only analytically continue the correlation functions (including the Green's function) to complex or imaginary κ.

Here we show that this prescription is advocated by the relation of the path-integral to quantum-mechanical transition amplitudes. Recall that after a spatial Fourier transformation a field can be considered as an infinite set of harmonic oscillators, each with action

$$S^{\text{bulk}} = \int_{t_0}^{t_f} dt \left[\frac{\dot{q}^2}{2} - \frac{\omega^2 q^2}{2}\right]. \tag{B.2}$$

This action is obtained from the quantum-mechanical transition amplitude

$$\int \mathcal{D}x e^{iS^{\text{bulk}}} = \langle x_N, t_f | e^{-i\hat{H}(t_f - t_0)} | x_1, t_0 \rangle\,, \qquad \hat{H} = \frac{\hat{p}^2}{2} + \omega^2 \frac{\hat{x}^2}{2}\,, \tag{B.3}$$

by splitting the interval $t_f - t_0$ into N smaller intervals of length $(t_f - t_0)/N$, inserting $N-1$ complete sets of $|x\rangle$ and N complete sets of $|p\rangle$ states, and taking the continuum limit $N \to \infty$. This derivation makes clear that the action (B.2) has boundary conditions $q(t_f) = x_N$, $q(t_0) = x_1$, and that the endpoints are *not* integrated over. Also clear is that temporal boundaries are quantum-mechanically on a very different footing than spatial boundaries. The latter simply affect the spatial modefunctions. Temporal boundaries, however, are encoded in the choice of initial and final state.

JHEP04(2004)076

For the free theory, a gaussian integral, the exact answer for the transition amplitude is easily obtained. One substitutes the solution to the field equation with boundary conditions $q(t_f) = x_N$, $q(t_0) = x_1$ into the action. Note that as the endpoints are not integrated over, the field equation is derived under the condition that the variation δq vanishes on the boundary, $\delta q(t_f) = 0$, $\delta q(t_i) = 0$. One finds the well-known results (up to normalizations, which we ignore throughout this appendix)

$$\begin{aligned} q_{\mathrm{sol}_1}(t) &= De^{i\omega t} + \text{c.c.}\ , \qquad D \equiv \frac{x_N e^{-i\omega t_0} - x_1 e^{-i\omega t_f}}{2i\sin(\omega(t_f - t_0))} \\ \int \mathcal{D}x\, e^{iS^{\mathrm{bulk}}} &= \exp\left[-\omega\left(\frac{D^2(e^{2i\omega t_f} - e^{2i\omega t_0})}{2} - \text{c.c.}\right)\right] \\ &\equiv e^{iS^{\mathrm{bg,\,bulk}}(x_N, x_1)}\,. \end{aligned} \tag{B.4}$$

Consider now the transition amplitude for a different initial state. In particular let us choose the harmonic oscillator vacuum $|0\rangle$ annihilated by $\hat{a} = \frac{1}{2}(i\hat{p} + \omega\hat{x})$. This corresponds to the Minkowski space vacuum for the field mode with frequency ω. The transition amplitude $\langle x_N|e^{-i\hat{H}(t_f - t_0)}|0\rangle$ can be obtained from the standard transition amplitude by the insertion of a complete set of states

$$\langle x_N|e^{-i\hat{H}(t_f-t_0)}|0\rangle = \int dx_1 \langle x_N|e^{-i\hat{H}(t_f-t_0)}|x_1\rangle\langle x_1|0\rangle\,. \tag{B.5}$$

We can evaluate this expression in two ways. Either we can substitute the harmonic oscillator ground state wave function $\langle x_1|0\rangle \simeq e^{-\omega x_1^2/2}$ and the result (B.4) for the propagator. Performing the remaining gaussian integral over x_1,

$$\int dx_1\, e^{iS^{\mathrm{bg,\,bulk}}(x_N,x_1)} e^{-\frac{\omega x_1^2}{2}} = e^{-\frac{\omega x_N^2}{2}}\,, \tag{B.6}$$

the result simply states that $|0\rangle$ is the zero energy eigenstate of the (normal-ordered) hamiltonian. Or we can again derive a path-integral by splitting the interval $t_f - t_0$ into N smaller intervals, now inserting N complete sets of $|x\rangle$ and N complete sets of $|p\rangle$ states, and taking the continuum limit $N \to \infty$. Doing so yields the bulk action (B.2) plus a boundary term

$$S^{\mathrm{bulk+bdy}} = \int_{t_0}^{t_f} dt \left[\frac{\dot{q}^2}{2} - \frac{\omega^2 q^2}{2}\right] - \kappa_0 \frac{q(t_0)^2}{2}\,. \tag{B.7}$$

As is clear from the ground state wave function $\langle x_1|0\rangle$ the boundary coupling κ_0 will be imaginary and equal to $\kappa_0 = -i\omega$. The Wick rotation intuition that the boundary couplings for spacelike boundaries are imaginary is confirmed. The answer for the transition amplitude $\langle x_N|e^{-i\hat{H}(t_f-t_0)}|0\rangle$ ought then follow from solving the field equations for this action including the boundary term, and substituting the solution back. The extra insertion $\int dx_1 |x_1\rangle\langle x_1|$ means that the endpoint $q(t_0)$ is now integrated over. The fluctuation $\delta q(t_0)$ therefore no longer vanishes and we obtain the field equations

$$\left(\frac{d^2}{dt^2} + \omega^2\right) q(t) = 0\,, \qquad \text{and} \qquad -\frac{d}{dt}q(t_0) - \kappa_0 q(t_0) = 0\,, \tag{B.8}$$

plus the implicit boundary condition $q(t_f) = x_N$.

Because the coordinate $q(t)$ is manifestly real, one has to give a prescription how to deal with the boundary condition (B.8) for imaginary κ. It is quite obvious that insisting on q real, i.e. $dq/dt(t_0) = 0 = q(t_0)$, or insisting that the action remain real, $q^2 \to |q|^2$, will not reproduce the known answer (B.6). However, if we simply proceed on the assumption that κ is real, i.e.

$$\begin{aligned} q_{sol_2}(t) &= \mathcal{A}(e^{i\omega t} + b_0 e^{-i\omega t})\,, \\ \mathcal{A}(e^{i\omega t_f} + b_0 e^{-i\omega t_f}) &= x_N\,, \qquad b_0 = -\frac{\kappa_0 + i\omega}{\kappa_0 - i\omega} e^{2i\omega t_0}\,, \end{aligned} \tag{B.9}$$

the answer for the background value of the action,

$$S^{\text{bg, bulk+bdy}} = \frac{i\omega}{2}\left(\frac{x_N^2}{(1+be^{-2i\omega t_f})^2} - \mathcal{A}^2 b^2 e^{-2i\omega t_f}\right), \tag{B.10}$$

precisely reproduces the answer (B.6) for $\kappa_0 = -i\omega$ (hence $b = 0$). This is therefore the prescription for dealing with imaginary boundary couplings: assume κ is real until the final answer, and only then analytically continue.

In the above example we have, of course, restricted ourselves to free field theory. One can repeat the whole exercise, however, with the inclusion of a bulk source term $iS \to iS + \int dt J(t)q(t)$ representing interactions. Treating the source perturbatively, we expand into fluctuations ξ around the background solution, $q(t) = q_{sol}(t) + \xi(t)$. Integrating the fluctuations out, we obtain for the action

$$S^{bulk+bdy}_{\kappa_f,\kappa_0}(q) = \int dt\left[\frac{\dot{q}^2}{2} - \omega^2\frac{q^2}{2} - iJq\right] + \kappa_f\frac{q(t_f)^2}{2} - \kappa_0\frac{q(t_0)^2}{2}\,, \tag{B.11}$$

the result

$$\begin{aligned} S^{\text{bg, bulk+bdy}}_{\kappa_f,\kappa_0}(J;q_{\text{sol}}) \;=\; & S^{\text{bg, bulk+bdy}}_{\kappa_f,\kappa_0}(0) - i\int dt J(t) q_{\text{sol}}(t) - \\ & -\frac{i}{2}\int dt dt'\, J(t) G_{\kappa_f,\kappa_0}(t,t') J(t')\,. \end{aligned} \tag{B.12}$$

where $G_{\kappa_f,\kappa_0}(t,t')$ is the Green's function of appendix A. Note that at endpoints where $q(t)$ is not integrated over, i.e. when $\delta q(t_{end})$ is constrained to vanish, $\xi(t_{end})$ also vanishes. At these points the Green's functions for the fluctuations ξ therefore obeys Dirichlet boundary conditions with $\kappa_{end} = \infty$. For the transition function $\langle x_N|e^{-i\hat{H}(t_f-t_0)}|x_1\rangle$ we thus have $\kappa_f = \infty = \kappa_0$, whereas for the transition function $\langle x_N|e^{-i\hat{H}(t_f-t_0)}|0\rangle$ we have $\kappa_f = \infty$, $\kappa_0 = -i\omega$. Equivalence between the two transistion functions including bulk sources is thus established if

$$\int dx_1 \exp\left[iS^{\text{bg, bulk+bdy}}_{\kappa_{f,0}=\infty}(J;q_{\text{sol}_1}(x_N,x_1))\right]\langle x_1|0\rangle = \exp\left[iS^{\text{bg, bulk+bdy}}_{\substack{\kappa_f=\infty,\\ \kappa_0=-i\omega}}(J;q_{\text{sol}_2}(x_N))\right]. \tag{B.13}$$

The only dependence on x_1 is in $q_{\text{sol}_1}(t)$ (eq. (B.4)). Using the Green's functions of appendix A, which are derived with the assumption that κ is analytic, it is an instructive exercise to verify that eq. (B.13) is indeed true. The prescription to deal with imaginary κ by analytic continuation to imaginary values in the final correlation functions, therefore holds for perturbation theory as well.

This example is an explicit manifestion of the fact that (in perturbation theory) all correlation functions are analytic in the coupling constants. This necessarily includes boundary couplings, which for a fixed time boundary correspond to initial conditions.

C. Boundary field redefinitions in the presence of irrelevant operators

We provide here the details behind the discontinuous shift of the field ϕ on the boundary which effectively sets the coefficients of the relevant and irrelevant operators $\oint \phi\partial_n\phi$, $\oint(\partial_n\phi)^2$ and $\oint \phi\partial_n^2\phi$ to zero.

After one integrates out high energy degrees of freedom, the most general form of the boundary action including the leading irrelevant boundary operators is

$$S_{\text{bound}} = \oint d^3x - \frac{\kappa}{2}\phi^2 - \frac{\mu}{2}\phi\partial_n\phi - \frac{\beta_{||}}{2M}\partial^i\phi\partial_i\phi - \frac{\beta_\perp}{2M}\partial_n\phi\partial_n\phi - \frac{\beta_c}{2M}\phi\partial_n^2\phi\,. \tag{C.1}$$

Let us focus on the last operator for a moment. It is well known that in effective field theories (bulk) irrelevant operators of dimension p containing the factor $\partial_t^2\phi$ can be removed by a field redefinition at the expense of introducing irrelevant operators of dimension $q > p$ [33, 34]. The only new element here is that the irrelevant operator is localized on the boundary. Generalizing, we see that the discontinuous field redefinition

$$\phi(y) \to \phi(y) + \delta(y_0 - y)\frac{\beta_c}{M}\phi(y) \tag{C.2}$$

precisely generates a term that cancels the coefficient of $\oint \phi\partial_n^2\phi$ to first order in β_c. To this same order the other couplings change as

$$\begin{aligned}
\kappa' &= \kappa + \frac{\beta_c}{M}(-m^2 + 2\kappa\delta(0) + \mu\delta'(0))\,,\\
\mu' &= \mu + \frac{\beta_c}{M}(2\mu - 2)\delta(0)\,,\\
\beta'_{||} &= \beta_{||} - \beta_c\,,\\
\beta'_\perp &= \beta_\perp\,,
\end{aligned} \tag{C.3}$$

with primed quantities denoting the effective value after the field redefinition (C.2). Here m^2 is the bulk mass. We have ignored any bulk contributions to the boundary action of order ϕ^3 and higher, having perturbation theory in mind. In appendix D we show that the explicit delta functions at zero argument, $\delta(0)$, serve to make all distributions conform to the boundary condition $\partial_n f(y) = -\kappa f(y)$.

To account for all couplings conflicting with the calculus of variations, μ, $\beta_{||}$, and β_c we combine the discontinuous field redefinition (C.2) with a discontinuous field redefinition of the form considered in section 2.

$$\phi(y) \to \phi(y) + \theta(y_0 - y)\left[\alpha_1\phi(y) + \alpha_2\partial_n\phi(y) + \cdots\right] + \delta(y_0 - y)\left[\widetilde{\alpha}\phi(y) + \cdots\right]. \tag{C.4}$$

We have left the coefficient $\widetilde{\alpha}$ arbitrary; as we will see there are additional compensations necessary beyond $\widetilde{\alpha} = \beta_c/M$. Note that both $\widetilde{\alpha}$ and α_2 have dimensions of M^{-1}. Consistent with the degree of approximation of the effective action, the field redefinition is an expansion in irrelevant terms to first order in M^{-1}.

Formally we can solve for α_i, $\widetilde{\alpha}$ in terms of μ, $\beta_\perp$ and β_c, so that the coefficients of the operators $\oint \phi\partial_n\phi$, $\oint(\partial_n\phi)^2$ and $\oint \phi\partial_n^2\phi$ vanish. The initial action $S_{\rm bound}$ of eq. (C.1) is therefore equal to an effective boundary action

$$S_{\rm eff} = \oint d^3x \, -\frac{\kappa_{\rm eff}(\alpha_i,\partial_i)}{2}\phi^2 \tag{C.5}$$

with the solutions for α_i substituted. (We absorbed the $\oint \beta_{||}\partial^i\phi\partial_i\phi$ into a momentum dependent $\kappa_{\rm eff}(\partial_i)$). At the end of the day we are only interested in the solution up to linear order in $\beta_\perp$, β_c. Higher order terms in $\beta_\perp$ and β_c would require the inclusion of higher order irrelevant operators for consistency. We may therefore linearize the problem and solve the system order by order in $\beta_\perp$, β_c. Substituting the zeroth and first order terms

$$\begin{aligned}
\alpha_1 &= \alpha_{10} + \alpha_{12}\frac{\beta_\perp}{M} + \alpha_{13}\frac{\beta_c}{M} + \mathcal{O}\left(\frac{\beta^2}{M^2}\right),\\
\alpha_2 &= 0 + \alpha_{22}\frac{\beta_\perp}{M} + \alpha_{23}\frac{\beta_c}{M} + \mathcal{O}\left(\frac{\beta^2}{M^2}\right),\\
\widetilde{\alpha} &= 0 + \widetilde{\alpha}_{42}\frac{\beta_\perp}{M} + \widetilde{\alpha}_{43}\frac{\beta_c}{M} + \mathcal{O}\left(\frac{\beta^2}{M^2}\right),
\end{aligned} \tag{C.6}$$

where α_{10} is the solution given in eq. (2.5), one finds (ignoring the zeroth order term in $\beta_\perp$, β_c)

$$\begin{aligned}
S_{\rm bound} &= S_{-1} + S_0 + S_{1_1} + S_{1_2}\,,\\
S_{1_2} &= \oint d^3x\,\phi\partial_n^2\phi\Bigg[\frac{\beta_\perp}{M}\left(\frac{-\alpha_{10}\alpha_{22}}{4} - \frac{\widetilde{\alpha}_{42}}{2} - \frac{\widetilde{\alpha}_{42}\alpha_{10}}{4} - \frac{\mu}{4}\alpha_{22}\left(1+\frac{\alpha_{10}}{2}\right)\right) + \\
&\qquad + \frac{\beta_c}{M}\left(\frac{-\alpha_{10}\alpha_{23}}{4} - \frac{\widetilde{\alpha}_{43}}{2} - \frac{\widetilde{\alpha}_{43}\alpha_{10}}{4} - \frac{\mu}{4}\alpha_{23}\left(1+\frac{\alpha_{10}}{2}\right) - \frac{1}{2}\left(1+\frac{\alpha_{10}}{2}\right)^2\right)\Bigg],\\
S_{1_1} &= \oint d^3x\,\partial_n\phi\partial_n\phi\Bigg[\frac{\beta_\perp}{M}\left(\frac{-\alpha_{22}}{2} - \frac{\alpha_{22}\alpha_{10}}{4} - \frac{\mu}{4}\alpha_{22}\left(1+\frac{\alpha_{10}}{2}\right) - \frac{1}{2}\left(1+\frac{\alpha_{10}}{2}\right)^2\right) + \\
&\qquad + \frac{\beta_c}{M}\left(-\frac{\alpha_{23}}{2} - \frac{\alpha_{23}\alpha_{10}}{4} - \frac{\mu}{4}\alpha_{23}\left(1+\frac{\alpha_{10}}{2}\right)\right)\Bigg],\\
S_0 &= \oint d^3x\,\phi\partial_n\phi\Bigg[\frac{\beta_\perp}{M}\Bigg(\frac{-\alpha_{12}}{2} - \frac{\alpha_{12}\alpha_{10}}{2} + \frac{\alpha_{10}\alpha_{22}}{2}\delta(0) - \frac{3H\widetilde{\alpha}_{42}}{2}\left(1+\frac{\alpha_{10}}{2}\right) + \\
&\qquad + \widetilde{\alpha}_{42}\left(1+\frac{3\alpha_{10}}{2}\right)\delta(0) - \frac{\kappa\alpha_{22}}{2}\left(1+\frac{\alpha_{10}}{2}\right) - \\
&\qquad - \frac{\mu}{2}\left((\alpha_{12}+2\widetilde{\alpha}_{42}\delta(0))\left(1+\frac{\alpha_{10}}{2}\right) - \alpha_{22}(1+\alpha_{10})\right) + \\
&\qquad + \alpha_{10}\left(1+\frac{\alpha_{10}}{2}\right)\delta(0)\Bigg) + \\
&\qquad + \frac{\beta_c}{M}\Bigg(\frac{-\alpha_{13}}{2} - \frac{\alpha_{13}\alpha_{10}}{2} + \frac{\alpha_{10}\alpha_{23}}{2}\delta(0) - \frac{3H\widetilde{\alpha}_{43}}{2}\left(1+\frac{\alpha_{10}}{2}\right) +
\end{aligned}$$

$$
\begin{aligned}
&+\widetilde{\alpha}_{43}\left(1+\frac{3\alpha_{10}}{2}\right)\delta(0)-\frac{\kappa\alpha_{23}}{2}\left(1+\frac{\alpha_{10}}{2}\right)- \\
&-\frac{\mu}{2}\Big((\alpha_{13}+2\widetilde{\alpha}_{43}\delta(0))\left(1+\frac{\alpha_{10}}{2}\right)-\alpha_{23}(1+\alpha_{10})\Big)+ \\
&+\alpha_{10}\left(1+\frac{\alpha_{10}}{2}\right)\delta(0)\Big)\Big],
\end{aligned}
$$

$$
\begin{aligned}
S_{-1} = \oint d^3x\,\phi^2 \Bigg[&\frac{\beta_\perp}{M}\Bigg(\frac{\alpha_{12}\alpha_{10}}{2}\delta(0)+\frac{\widetilde{\alpha}_{42}}{2}\hat{m}^2\left(1+\frac{\alpha_{10}}{2}\right)+\frac{\widetilde{\alpha}_{42}\alpha_{10}}{2}\delta'(0)-\widetilde{\alpha}_{42}\alpha_{10}\delta^2(0)+ \\
&+\frac{3H\widetilde{\alpha}_{42}\alpha_{10}}{2}\delta(0)-\kappa\left(\left(\frac{\alpha_{12}}{2}+\widetilde{\alpha}_{42}\delta(0)\right)\left(1+\frac{\alpha_{10}}{2}\right)\right)- \\
&-\frac{\mu}{2}\Big(-\left(\frac{\alpha_{12}}{2}+\widetilde{\alpha}_{42}\delta(0)\right)\alpha_{10}\delta(0)+(-\alpha_{12}\delta(0)+\widetilde{\alpha}_{42}\delta'(0))\left(1+\frac{\alpha_{10}}{2}\right)\Big)- \\
&-\frac{\alpha_{10}^2}{2}\delta^2(0)\Bigg)+ \\
&+\frac{\beta_c}{M}\Bigg(\frac{\alpha_{13}\alpha_{10}}{2}\delta(0)+\frac{\widetilde{\alpha}_{43}}{2}\hat{m}^2\left(1+\frac{\alpha_{10}}{2}\right)+\frac{\widetilde{\alpha}_{43}\alpha_{10}}{2}\delta'(0)-\widetilde{\alpha}_{43}\alpha_{10}\delta^2(0)+ \\
&+\frac{3H\widetilde{\alpha}_{43}\alpha_{10}}{2}\delta(0)-\kappa\left(\left(\frac{\alpha_{13}}{2}+\widetilde{\alpha}_{43}\delta(0)\right)\left(1+\frac{\alpha_{10}}{2}\right)\right)- \\
&-\frac{\mu}{2}\Big(-\left(\frac{\alpha_{13}}{2}+\widetilde{\alpha}_{43}\delta(0)\right)\alpha_{10}\delta(0)+(-\alpha_{13}\delta(0)+\widetilde{\alpha}_{43}\delta'(0))\left(1+\frac{\alpha_{10}}{2}\right)\Big)+ \\
&+\frac{\alpha_{10}}{2}\left(1+\frac{\alpha_{10}}{2}\right)\delta'(0)\Bigg)\Bigg].
\end{aligned}
\tag{C.7}
$$

These equations can be explicitly solved (e.g. $\alpha_{23}=0$). For the case $\mu=0$ (as in Bunch-Davies for instance), and hence $\alpha_{10}=0$, the solutions are easily found: $\alpha_{23}=0$, $\alpha_{22}=-1$, $\widetilde{\alpha}_{42}=0$, $\widetilde{\alpha}_{43}=-1$, $\alpha_{13}=(2\delta(0)-3H)$, $\alpha_{12}=\kappa$, with the answer for S_{-1}:

$$
S_{-1}=\oint d^3x\;-\frac{\phi^2}{2}\left[\kappa+\frac{\beta_\perp}{M}\kappa^2-\frac{\beta_c}{M}\left(m^2+k^2-3H\kappa\right)+4\kappa\frac{\beta_c}{M}\delta(0)\right]. \tag{C.8}
$$

As we will show in the next appendix, the term $\kappa\beta_c\delta(0)$ solely served to make all distributions consistent with the boundary condition $\partial_n f(y)=-\kappa f(y)$. We may therefore drop this term, as long as we remember this.

D. Distributions, boundary conditions and the equivalence between perturbation field theory and field redefinitions

The result (C.8) for the effective boundary action after the field redefinition suggests that the correction to the two-point function due to irrelevant operators contains delta functions at zero argument. In the perturbative Feynman diagram approach to the two-point function, which we perform in the next appendix, we shall find no explicit $\delta(0)$ terms. Yet the two approaches are manifestly equivalent, so somehow a further step is needed in the field redefinition approach to explain why no $\delta(0)$ term arises in the two-point correlator.

To understand the equivalence between the two approaches better, consider a matrix integral simplification of the path integral

$$
\langle x^k x^l\rangle=\int dx^i x^k x^l e^{-\frac{A_{ij}x^ix^j}{2}-\frac{\kappa_{ij}x^ix^j}{2}-\frac{\beta_{ij}x^ix^j}{2}}\,. \tag{D.1}
$$

Here κ_{ij} and β_{ij} correspond to the boundary interactions; whereas A_{ij} is the kinetic operator. We will now evaluate this integral in two ways (1) by a saddlepoint approximation, i.e. a Feynman diagram expansion with β_{ij} treated as an interaction, and (2) by a field redefinition which absorbs β_{ij} at the expense of redefining κ_{ij}. Expanding the answer (2) to linear order in β_{ij} we should reobtain the Feynman diagram result.

The Feynman diagram approach: expanding to linear order in β_{ij} we find that

$$\begin{aligned}\langle x^k x^l\rangle &= \int dx^i x^k x^l \left(1-\frac{\beta_{ij}x^i x^j}{2}+\cdots\right) e^{-\frac{(A+\kappa)_{ij}}{2}x^2} \\ &= N\frac{\partial}{\partial J_k}\frac{\partial}{\partial J_l}\left(1-\frac{\beta_{ij}}{2}\frac{\partial}{\partial J_i}\frac{\partial}{\partial J_j}\right) e^{\frac{1}{2}J_i G_\kappa^{ij} J_j}\Big|_{J=0}\,. \end{aligned} \tag{D.2}$$

Here N is an unimportant normalization, and we have introduced the Green's function $G_\kappa = (A+\kappa)^{-1}$.[27] The gaussian integrals are easily evaluated to

$$\langle x^k x^l\rangle = G_\kappa^{kl} - \beta_{ij}G_\kappa^{jl}G_\kappa^{ik} + \left(1-\frac{\beta_{ij}}{2}G_\kappa^{ij}\right)G_\kappa^{lk}\,. \tag{D.3}$$

We clearly recognize the connected and loop diagrams.

Field redefinitions: the field redefinition is designed such that to first order the contribution from the kinetic part cancels the β_{ij} factor. Hence

$$x^i \to x^i - \frac{G^{ij}\beta_{jk}}{2}x^k = \left(1-\frac{G\beta}{2}\right)x \equiv Sx\,, \tag{D.4}$$

where we have introduced a second Green's function $G \equiv A^{-1}$. Note that $G \neq G_\kappa$. Under this field redefinition the integral becomes

$$\langle x^k x^l\rangle = S^k_p S^l_q \int dx\, |\mathrm{Jac}|\, x^p x^q e^{-\frac{x^\top S^\top (A+\kappa+\beta)Sx}{2}}\,. \tag{D.5}$$

The jacobian $|\mathrm{Jac}|$ will contain the loop diagrams. Our interest only extends to connected diagrams and we may therefore ignore it. Expanding to linear order in β_{ij} we get

$$\langle x^k x^l\rangle = S^k_p S^l_q \int dx\, |\mathrm{Jac}|\, x^p x^q e^{-\frac{1}{2}x^\top\left(A+\kappa-\frac{\kappa G\beta}{2}-\frac{\beta^\top G\kappa}{2}\right)x}\,. \tag{D.6}$$

The term proportional to $\kappa\beta$ is exactly the problematic one, as we will see. Thus the two-point function is easily evaluated to

$$\begin{aligned}\langle x^k x^l\rangle &= S^k_p S^l_q \left(G_\kappa + G_\kappa\left(\frac{\kappa G\beta + \beta^\top G\kappa}{2}\right)G_\kappa\right)^{pq} \\ &= \left(G_\kappa - \frac{G\beta G_\kappa}{2} - \frac{G_\kappa\beta^\top G}{2} + G_\kappa\left(\frac{\kappa G\beta+\beta^\top G\kappa}{2}\right)G_\kappa\right)^{kl} \end{aligned} \tag{D.7}$$

$$\begin{aligned} &= \left(G_\kappa - \frac{G_\kappa}{2}\Big((A+\kappa)G\beta - \kappa G\beta - \beta^\top G\kappa + \beta^\top G(A+\kappa)\Big)G_\kappa\right)^{kl} \\ &= \left(G_\kappa - G_\kappa\beta G_\kappa\right)^{kl}\,. \end{aligned} \tag{D.8}$$

[27] In the following we will use matrix and index notation interchangeably. It should be clear from the context which notation is being used.

In the second to last step we recalled that $G_\kappa = (A+\kappa)^{-1}$. We see that we exactly reproduce the connected diagrams as expected.

Applying these lessons to field redefinitions on the boundary: as eq. (D.7) shows field redefinitions which are localized on the boundary ought to have no effect on bulk correlators. This means that the fourth term in (D.7) ought to reproduce the Feynman diagram computation. If the $\kappa G \beta$ factor contains the $\delta(0)$ term, this appears not to be the case. The resolution follows from repeating the steps (D.7) in detail in the field theory.

If we consider the index i as the location in the y direction, we easily see that in the (free) field theory of section 2 with boundary interaction (C.1) and $\beta_\perp = \beta_\| = 0$ the matrices A, κ, β correspond to the differential operators.

$$\begin{aligned} A_{ij} = A(y_1,y_2) &= \delta(y_1-y_2)\partial_1\partial_2\,, \\ \kappa &= \kappa_{co}\delta(y_0-y_1)\delta(y_1-y_2)\,, \\ \beta &= \beta_{co}\delta(y_1-y_2)\delta(y_0-y_1)\Box_2\,. \end{aligned} \tag{D.9}$$

We have given the couplings a subscript *co* to distinguish them from the matrix operators. We easily compute that

$$\begin{aligned} A^{-1}(y_1,y_2) &= G(y_1,y_2) \qquad \text{with } \Box G = -\delta(y_1-y_2) \quad \text{and} \quad \partial_y G = 0 \\ G\beta &= \beta_{co}\int dy_2 G(y_1,y_2)\Box_2\delta(y_2-y_3)\delta(y_0-y_2) \\ &= -\beta_{co}\delta(y_1-y_3)\delta(y_0-y_3)\,. \end{aligned} \tag{D.10}$$

The transformation S is thus indeed the one we consider in eq. (C.2).

$$\begin{aligned} \int dy_2 S(y_1,y_2)\phi(y_2) &= \int dy_2\delta(y_1-y_2)\phi(y_2) + \frac{\beta_{co}}{2}\delta(y_1-y_2)\delta(y_0-y_2)\phi(y_2) \\ &= \phi(y_1) + \frac{\beta_{co}}{2}\delta(y_0-y_1)\phi(y_0)\,. \end{aligned} \tag{D.11}$$

We also see that

$$\kappa G\beta = -\kappa_{co}\beta_{co}\delta(y_0-y_1)\delta(y_1-y_2)\delta(y_0-y_2) \tag{D.12}$$

and hence that it contains the problematic $\delta(0)$ term:

$$\int dy_1 dy_2 \phi(y_1)\phi(y_2)\left[\kappa G\beta\right](y_1,y_2) = -\kappa_{co}\beta_{co}\phi(y_0)^2\delta(0)\,. \tag{D.13}$$

Because this $\delta(0)$ term is present in the action, we expect it to be present in the two-point correlator as well. Indeed a straightforward computation gives (Note that $\hat{A}_\kappa^{-1} = G_\kappa$: the Green's function obeying the boundary condition $\partial_y G = -\kappa G$.)

$$\begin{aligned} \langle\phi(y_1)\phi(y_2)\rangle &= G_\kappa(y_1,y_2) + \delta(y_1-y_0)\frac{G_\kappa(y_0,y_2)}{2}\beta_{co} + \beta_{co}\frac{G_\kappa(y_1,y_0)}{2}\delta(y_0-y_2) \\ &\quad - G_\kappa(y_1,y_0)\delta(0)G_\kappa(y_0,y_2)\,. \end{aligned} \tag{D.14}$$

The Feynman diagram computation, however, has no $\delta(0)$. How can this agree? We have seen the explicit steps we need to do to get the Feynman diagram answer. Surprisingly when we implement them here, the $\delta(0)$ cancels. We will need that

$$\begin{aligned} A+\kappa &= \delta(y_1-y_2)\partial_1\partial_2+\kappa_{co}\delta(y_0-y_1)\delta(y_1-y_2) \\ &\simeq -\delta(y_1-y_2)\Box_1+\delta(y_1-y_2)\delta(y_0-y_1)(\partial_1-\kappa)\,. \end{aligned} \tag{D.15}$$

Its inverse, the Green's function G_κ, obeys a slightly different differential equation, however. As we also discussed in appendix A, acting with the laplacian on G_κ returns the delta function $\delta_\kappa(y_1-y_2)$ in the space of functions obeying $\partial_y f(y_0) = -\kappa f(y_0)$. There are additional contributions from image charges which guarantee that on the boundary $\partial_y\delta_\kappa = -\kappa\delta_\kappa$. Now repeating the steps from eq. (D.7)

$$\begin{aligned} \langle\phi(y_1)\phi(y_2)\rangle &= G_\kappa(y_1,y_2) \\ &\quad+\int dy_3dy_4\beta_{co}G_\kappa(y_1,y_3)(-\Box_3+\kappa_{co}\delta(y_0-y_3))\times \\ &\qquad\times\delta(y_3-y_4)\delta(y_4-y_0)\frac{G_\kappa(y_0,y_2)}{2}\beta_{co}+ \\ &\quad+\int dy_3dy_4\beta_{co}\frac{G_\kappa(y_1,y_0)}{2}\delta(y_0-y_4)(-\Box_3+\kappa_{co}\delta(y_0-y_3))\times \\ &\qquad\times\delta(y_4-y_3)G_\kappa(y_3,y_2)-G_\kappa(y_1,y_0)\delta(0)G_\kappa(y_0,y_2) \\ &= G_\kappa(y_1,y_2)-\frac{\beta_{co}}{2}\delta_\kappa(y_1-y_0)G_\kappa(y_0,y_2)-\frac{\beta_{co}}{2}\delta_\kappa(y_2-y_0)G_\kappa(y_1,y_0)\,, \end{aligned} \tag{D.16}$$

we recognize that the sole function of the $\delta(0)$ term in the action is to correctly implement the boundary conditions for the transformation $S = 1 - G\beta/2$. Indeed it is clear from the matrix analogue that had we started with a transformation $S_\kappa = (1 - G_\kappa\beta/2)$ no distributions at zero argument $\delta(0)$ would have been generated at all.

The distribution $\delta_\kappa(y_1-y_0)$ with the correct boundary conditions which thus appears, has no support deep in the bulk, $\delta_\kappa(y_1-y_0)$ vanishes for $y_1 \gg y_0$ of course. The lesson we extract from this exercise is that *for bulk correlators we may ignore the $\delta(0)$ term in the action.*

Other field redefinitions: if field redefinitions $\phi(y)\to\phi(y)+\delta(y-y_0)\widetilde{\alpha}\phi(y_0)$ to remove the irrelevant operator $\oint\beta_{co}\phi\partial_n^2\phi$ leave no trace in bulk correlators, an obvious question is why the "theta" transformations do contribute. They do, and why follows from repeating the above steps for that case. Consider for simplicity only the relevant correction μ. In the above language it corresponds to choosing

$$\beta_\mu = \mu_{co}\delta(y_1-y_0)\delta(y_1-y_2)\partial_{y_1}\,. \tag{D.17}$$

Therefore

$$G\beta_\mu = \int dy_2\partial_{y_2}G(y_1,y_2)\delta(y_2-y_0)\delta(y_2-y_3) = \partial_{y_3}G(y_1,y_3)\delta(y_3-y_0)\,. \tag{D.18}$$

Now (in Minkowski space) we can show that this is exactly the step function transformation. Upon use of the identity

$$G\beta_\mu = -\partial_{y_1}G(y_1,y_2)\delta(y_2-y_0)\,, \tag{D.19}$$

we can take one more derivative to obtain

$$-\partial_1^2 G(y_1,y_2)\delta(y_2-y_0) = -\delta(y_1-y_2)\delta(y_2-y_0)\,. \tag{D.20}$$

Thus $G\beta_\mu$ precisely has the property that it's derivative is the delta function — it is therefore proportional to the θ function. G, moreover, is the Neumann Green's function. Hence $G\beta$ is zero in the bulk, it is precisely equal to $\theta(y_0-y_1)$.

Because $\theta(y_0-y_1)$ is of measure zero, the statement that the second and third terms $G\beta_\mu G_\kappa^{-1}$ arising from the explicit field redefinition do not contribute to the bulk, is now manifest. Thus the fourth term — the one that comes directly from the action — ought to reproduce the Feynman diagram result. Indeed it is easy to see that

$$\int dy_2 dy_5 G_\kappa(y_1,y_2)(-\kappa_{co}\mu_{co}\delta(y_2-y_0)\partial G(y_2,y_5)\delta(y_5-y_0))G_\kappa(y_5,y_4) \tag{D.21}$$

precisely reproduces the perturbative Feynman diagram calculation, when we use the just derived result that $\partial G(y_0,y_0)=\theta(0)=1/2$.

E. Power spectrum corrections from perturbation theory

Aside from using field redefinitions on the boundary, one can also use field theory perturbation theory to compute the corrections to the power spectrum. For completeness we give that calculation here. The answer is, of course, the same as in eq. (4.19) to first order in β_i.

The first order correction to the (connected) two-point correlation function by a generalized two-point vertex

$$S^{\rm int} = -\int d^4x\sqrt{-g}\frac{\lambda}{2}\phi^2 = -\int d^4x_3 d^4x_4\sqrt{-g(x_3)}\sqrt{-g(x_4)}\frac{\lambda(x_3,x_4)}{2}\phi(x_3)\phi(x_4) \tag{E.1}$$

is

$${}_{\kappa_f}\langle\phi(x_1)\phi(x_2)\rangle_\kappa = -i\int d^4x_3 d^4x_4\sqrt{-g(x_3)}\sqrt{-g(x_4)}\,\lambda(x_3,x_4)G_\kappa(x_1,x_3)G_\kappa(x_2,x_4)\,. \tag{E.2}$$

Here κ_f, κ denote the future-out and past-in state and $G_\kappa(x_1,x_2)$ is therefore the Green's function satisfying $(\Box_1-m^2)G_\kappa(x_1,x_2)=i\delta^4_\kappa(x_1-x_2)/\sqrt{-g}$ with the boundary conditions

$$\begin{aligned} a_0^{-1}\partial_{\eta_1}G_\kappa(x_1,x_2)|_{\eta_1=\eta_0} &= -\kappa G_\kappa(x_1,x_2)|_{\eta_1=\eta_0} \\ \lim_{\eta_f\to\infty} a_f^{-1}\partial_{\eta_1}G_\kappa(x_1,x_2)|_{\eta_1=\eta_f} &= -\kappa_f G_\kappa(x_1,x_2)|_{\eta_1=\eta_f}\,. \end{aligned} \tag{E.3}$$

For the boundary interaction due to the leading boundary irrelevant operators (4.6), the (spacetime dependent) coupling $\lambda(x_3,x_4)$ considered as a derivative operator equals

$$\begin{aligned} \lambda(x_3,x_4) = {}& 2\left(\frac{\delta(\eta_3-\eta_0)}{a(\eta_3)}\frac{\delta^3(x_3-x_4)}{a^3(\eta_3)}\frac{\delta(\eta_3-\eta_4)}{a(\eta_3)}\right)\times \\ &\times\left[\frac{\beta_{||}}{a^2(\eta_3)M}\partial^{x_3,i}\partial_i^{x_4}+\frac{\beta_\perp}{a^2(\eta_3)M}\partial_{\eta_3}\partial_{\eta_4}+\right. \\ &\left.+\frac{\beta_c}{2a^2(\eta_3)M}\left(D_{\eta_4}\partial_{\eta_4}+D_{\eta_3}\partial_{\eta_3}\right)+\frac{\mu}{2a(\eta_3)}\left(\partial_{\eta_3}+\partial_{\eta_4}\right)\right]. \end{aligned} \tag{E.4}$$

(We purposely avoid integrating by parts, because $\lambda(x_3, x_4)$ arises from a boundary action rewritten as bulk interactions. Integration by parts would make this origin less clear.) Inserting this expression and the appropriate FRW quantities in eq. (E.2), we obtain after a spatial Fourier transform

$$
\begin{aligned}
&{}_{\kappa_f}\langle\phi(\eta_1,\vec{k}_1)\phi(\eta_2,\vec{k}_2)\rangle_\kappa = \\
&= -2i \oint_{\eta_3=\eta_4=\eta_0} d^3x_3 d^3x_1 d^3x_2\, a_0^3 e^{-i\vec{k}_1 x_1 - i\vec{k}_2 x_2} \int \frac{d^3\vec{k}_3 d^3\vec{k}_4}{(2\pi)^6} \times \\
&\quad \times \left[-\vec{k}_3\cdot\vec{k}_4 \frac{\beta_{||}}{a_0^2 M} + \frac{\beta_\perp}{a_0^2 M}\partial_{\eta_3}\partial_{\eta_4} + \frac{\beta_c}{2M}\left(-\Box_3 - \Box_4 - \frac{3H}{a}(\partial_{\eta_3}+\partial_{\eta_4}) - \frac{\vec{k}_3^2+\vec{k}_4^2}{a_0^2} \right) + \right. \\
&\quad \left. + \frac{\mu}{2a_0}(\partial_{\eta_3}+\partial_{\eta_4}) \right] G_\kappa(\vec{k}_3,\eta_1,\eta_3) G_\kappa(\vec{k}_4,\eta_2,\eta_4) e^{i\vec{k}_3(x_1-x_3)+i\vec{k}_4(x_2-x_3)} \\
&= -2i(2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2)a_0^3 \times \\
&\quad \times \left[\frac{\vec{k}_1^2\beta_{||}}{a_0^2 M} + \frac{\beta_\perp}{a_0^2 M}\partial_{\eta_3}\partial_{\eta_4} + \frac{\beta_c}{M}\left(-\frac{1}{2}\Box_3 - \frac{1}{2}\Box_4 - \frac{3H}{2a_0}(\partial_{\eta_3}+\partial_{\eta_4}) - \frac{\vec{k}_1^2}{a_0^2} \right) + \right. \\
&\quad \left. + \frac{\mu}{2a_0}(\partial_{\eta_3}+\partial_{\eta_4}) \right] G_\kappa(\vec{k}_1,\eta_1,\eta_3) G_\kappa(\vec{k}_1,\eta_2,\eta_4)\Big|_{\eta_3=\eta_4=\eta_0} \\
&= -2i(2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2)a_0^3 \times \\
&\quad \times \left[\frac{\vec{k}_1^2\beta_{||}}{a_0^2 M} + \frac{\kappa^2\beta_\perp}{M} + \frac{\beta_c}{M}\left(-i\frac{\delta_\kappa(\eta_1-\eta_0)+\delta_\kappa(\eta_2-\eta_0)}{2a_0^4} - m^2 - \frac{\vec{k}_1^2}{a_0^2} + 3H\kappa \right) - \right. \\
&\quad \left. - \mu\kappa \right] G_\kappa(\vec{k}_1,\eta_1,\eta_0) G_\kappa(\vec{k}_1,\eta_2,\eta_0)\,.
\end{aligned}
\tag{E.5}
$$

In the first step we related the double normal derivative $D_n\partial_n$ to the laplacian. In the second step we used both defining property of the Green's function $(\Box - m^2)G_\kappa = i\delta_\kappa^4/a^4$ *and* the boundary condition $\partial_\eta G_\kappa = -a_0\kappa G_\kappa$. Recall the expression for $G_\kappa(\vec{k}_1,\eta_1,\eta_2)$ from eq. (3.7) in terms of the basis functions $\phi_{\mathrm{dS},\pm}$. For the power spectrum we are interested in the equal time two-point correlator for $\eta_1 = \eta_2 \to 0$. In that limit the Green's function only retains the retarded contribution

$$
G_\kappa(\vec{k}_1,\eta_1,\eta_0)_{\eta_1\gg\eta_0} = \overline{\varphi}_{b_{\kappa_f}}(\eta_1)\varphi_{b_\kappa}(\eta_0)\,. \tag{E.6}
$$

Below we shall see that for the inflationary power spectrum, we should choose $\kappa_f = \bar{\kappa}$. The equal-time correlator at $\eta_1 = \eta_2 \to 0$ therefore equals

$$
\begin{aligned}
\lim_{\eta_1\to 0} {}_\kappa\langle\phi(\eta_1,\vec{k}_1)\phi(\eta_1,\vec{k}_2)\rangle_\kappa &= -2i(2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2)a_0^3\overline{\varphi}^2_{b_\kappa}(\eta_1)\varphi^2_{b_\kappa}(\eta_0) \times \\
&\times \left[\frac{\vec{k}_1^2(\beta_{||}-\beta_c)}{a_0^2 M} + \frac{\kappa^2\beta_\perp}{M} - \frac{\beta_c m^2}{M} - \kappa(\mu - \frac{3\beta_c H}{M}) \right].
\end{aligned}
\tag{E.7}
$$

Using the proportionality relation (4.13) between the basis-functions φ_b and $\overline{\varphi}_b$ as $\eta \to 0$, plus the expression for the zeroth order two-point correlator we obtain

$$\lim_{\eta_1\to 0} {}_\kappa\langle\phi(\eta_1,\vec{k}_1)\phi(\eta_1,\vec{k}_2)\rangle_\kappa = \langle\phi^2\rangle_0\left(\left(\frac{1-\overline{b}}{b-1}\right)(-2ia_0^3)\varphi_{b_\kappa}(\eta_0)^2 \times \right. \tag{E.8}$$
$$\left. \times\left[\frac{\vec{k}_1^2(\beta_{||}-\beta_c)}{a_0^2 M}+\frac{\kappa^2\beta_\perp}{M}-\frac{\beta_c m^2}{M}-\kappa\left(\mu-\frac{3\beta_c H}{M}\right)\right]\right).$$

Finally substituting the explicit expressions for φ_{b_κ},

$$\lim_{\eta_1\to 0} {}_\kappa\langle\phi(\eta_1,\vec{k}_1)\phi(\eta_1,\vec{k}_2)\rangle_\kappa = \langle\phi^2\rangle_0\left(\left(\frac{1-\overline{b}}{b-1}\right)\frac{-2i\pi}{4H}\overline{H}^2_{b,\nu}(-\vec{k}\eta_0) \times \right. \tag{E.9}$$
$$\left. \times\left[\frac{\vec{k}_1^2(\beta_{||}-\beta_c)}{a_0^2 M}+\frac{\kappa^2\beta_\perp}{M}-\frac{\beta_c m^2}{M}-\kappa\left(\mu-\frac{3\beta_c H}{M}\right)\right]\right).$$

with the obvious shorthand $H_{b,\nu}=H_\nu+b\overline{H}_\nu$.

The power spectrum of inflationary density perturbations due to spontaneous pair production in a gravitational background is obtained by the optical theorem from the two-particle cut of the one-loop vacuum amplitude $\langle\kappa|\kappa\rangle$.

$$P(\vec{k})\frac{d|\vec{k}|}{|\vec{k}|}=\frac{(4\pi)|\vec{k}|^3}{(2\pi)^3}\lim_{\eta_1\to 0}\mathrm{Im}\left(\frac{{}_\kappa\langle\phi(\eta_1,\vec{k}_1)\phi(\eta_1,\vec{k}_2)\rangle_\kappa}{-i}\right)\frac{d|\vec{k}|}{|\vec{k}|}. \tag{E.10}$$

This shows that $\kappa_f=\bar{\kappa}$. (Note the factor of i; this is a consequence of our normalization for the Green's function.) The imaginary part of the (Feynman time-ordered) Green's function is also known as the Wightman function. In contrast to the Green's function, the latter is a homogeneous solution to the field equation. We thus find

$$P_{\kappa+\delta\beta} = P_\kappa\left(\frac{\pi}{4H}\left[\left(\frac{1-\overline{b}}{b-1}\right)\frac{\overline{H}^2_{b,\nu}(-\vec{k}\eta_0)}{i}\left[\frac{\vec{k}_1^2(\beta_{||}-\beta_c)}{a_0^2M}+\frac{\kappa^2\beta_\perp}{M}\right.\right.\right.$$
$$\left.\left.\left.-\frac{\beta_c m^2}{M}-\kappa\left(\mu-\frac{3\beta_c H}{M}\right)\right]+\text{c.c.}\right]\right) \tag{E.11}$$

which agrees with eq. (4.19).

References

[1] R.H. Brandenberger, *Inflationary cosmology: progress and problems*, hep-ph/9910410;
J. Martin and R.H. Brandenberger, *The trans-planckian problem of inflationary cosmology*, *Phys. Rev.* **D 63** (2001) 123501 [hep-th/0005209];
R.H. Brandenberger and J. Martin, *The robustness of inflation to changes in super–Planck-scale physics*, *Mod. Phys. Lett.* **A 16** (2001) 999 [astro-ph/0005432];
J.C. Niemeyer, *Inflation with a high frequency cutoff*, *Phys. Rev.* **D 63** (2001) 123502 [astro-ph/0005533].

JHEP04(2004)076

[2] R. Easther, B.R. Greene, W.H. Kinney and G. Shiu, *Inflation as a probe of short distance physics*, *Phys. Rev.* **D 64** (2001) 103502 [hep-th/0104102];
R. Easther, B.R. Greene, W.H. Kinney and G. Shiu, *Imprints of short distance physics on inflationary cosmology*, *Phys. Rev.* **D 67** (2003) 063508 [hep-th/0110226];
R. Easther, B.R. Greene, W.H. Kinney and G. Shiu, *A generic estimate of trans-planckian modifications to the primordial power spectrum in inflation*, *Phys. Rev.* **D 66** (2002) 023518 [hep-th/0204129];
C.-S. Chu, B.R. Greene and G. Shiu, *Remarks on inflation and noncommutative geometry*, *Mod. Phys. Lett.* **A 16** (2001) 2231 [hep-th/0011241].

[3] G. Shiu and I. Wasserman, *On the signature of short distance scale in the cosmic microwave background*, *Phys. Lett.* **B 536** (2002) 1 [hep-th/0203113].

[4] N. Kaloper, M. Kleban, A.E. Lawrence and S. Shenker, *Signatures of short distance physics in the cosmic microwave background*, *Phys. Rev.* **D 66** (2002) 123510 [hep-th/0201158];
N. Kaloper, M. Kleban, A. Lawrence, S. Shenker and L. Susskind, *Initial conditions for inflation*, *J. High Energy Phys.* **11** (2002) 037 [hep-th/0209231].

[5] J. Martin and R.H. Brandenberger, *A cosmological window on trans-planckian physics*, astro-ph/0012031;
T. Tanaka, *A comment on trans-planckian physics in inflationary universe*, astro-ph/0012431;
A. Kempf, *Mode generating mechanism in inflation with cutoff*, *Phys. Rev.* **D 63** (2001) 083514 [astro-ph/0009209];
J.C. Niemeyer and R. Parentani, *Trans-planckian dispersion and scale-invariance of inflationary perturbations*, *Phys. Rev.* **D 64** (2001) 101301 [astro-ph/0101451];
A. Kempf and J.C. Niemeyer, *Perturbation spectrum in inflation with cutoff*, *Phys. Rev.* **D 64** (2001) 103501 [astro-ph/0103225];
A.A. Starobinsky, *Robustness of the inflationary perturbation spectrum to trans-planckian physics*, *Zh. Eksp. Teor. Fiz.* **73** (2001) 415 [astro-ph/0104043];
M. Bastero-Gil, *What can we learn by probing trans-planckian physics*, hep-ph/0106133;
L. Hui and W.H. Kinney, *Short distance physics and the consistency relation for scalar and tensor fluctuations in the inflationary universe*, *Phys. Rev.* **D 65** (2002) 103507 [astro-ph/0109107];
M. Lemoine, M. Lubo, J. Martin and J.-P. Uzan, *The stress-energy tensor for trans-planckian cosmology*, *Phys. Rev.* **D 65** (2002) 023510 [hep-th/0109128];
R.H. Brandenberger, S.E. Joras and J. Martin, *Trans-planckian physics and the spectrum of fluctuations in a bouncing universe*, *Phys. Rev.* **D 66** (2002) 083514 [hep-th/0112122];
J. Martin and R.H. Brandenberger, *The Corley-Jacobson dispersion relation and trans-planckian inflation*, *Phys. Rev.* **D 65** (2002) 103514 [hep-th/0201189];
R. Brandenberger and P.-M. Ho, *Noncommutative spacetime, stringy spacetime uncertainty principle and density fluctuations*, *Phys. Rev.* **D 66** (2002) 023517 [hep-th/0203119];
S. Shankaranarayanan, *Is there an imprint of Planck scale physics on inflationary cosmology?*, *Class. and Quant. Grav.* **20** (2003) 75 [gr-qc/0203060];
S.F. Hassan and M.S. Sloth, *Trans-planckian effects in inflationary cosmology and the modified uncertainty principle*, *Nucl. Phys.* **B 674** (2003) 434 [hep-th/0204110];
J.C. Niemeyer, R. Parentani and D. Campo, *Minimal modifications of the primordial power spectrum from an adiabatic short distance cutoff*, *Phys. Rev.* **D 66** (2002) 083510 [hep-th/0206149];

K. Goldstein and D.A. Lowe, *Initial state effects on the cosmic microwave background and trans-planckian physics*, *Phys. Rev.* **D 67** (2003) 063502 [hep-th/0208167];
V. Bozza, M. Giovannini and G. Veneziano, *Cosmological perturbations from a new-physics hypersurface*, *JCAP* **05** (2003) 001 [hep-th/0302184];
G.L. Alberghi, R. Casadio and A. Tronconi, *Trans-planckian footprints in inflationary cosmology*, *Phys. Lett.* **B 579** (2004) 1 [gr-qc/0303035];
J. Martin and R. Brandenberger, *On the dependence of the spectra of fluctuations in inflationary cosmology on trans-planckian physics*, *Phys. Rev.* **D 68** (2003) 063513 [hep-th/0305161];
N. Kaloper and M. Kaplinghat, *Primeval corrections to the CMB anisotropies*, *Phys. Rev.* **D 68** (2003) 123522 [hep-th/0307016].

[6] U.H. Danielsson, *A note on inflation and transplanckian physics*, *Phys. Rev.* **D 66** (2002) 023511 [hep-th/0203198]; *Inflation, holography and the choice of vacuum in de Sitter space*, *J. High Energy Phys.* **07** (2002) 040 [hep-th/0205227].

[7] C.P. Burgess, J.M. Cline, F. Lemieux and R. Holman, *Are inflationary predictions sensitive to very high energy physics?*, *J. High Energy Phys.* **02** (2003) 048 [hep-th/0210233];
C.P. Burgess, J.M. Cline and R. Holman, *Effective field theories and inflation*, *JCAP* **10** (2003) 004 [hep-th/0306079];
C.P. Burgess, J. Cline, F. Lemieux and R. Holman, *Decoupling, trans-planckian and inflation*, astro-ph/0306236.

[8] C.L. Bennett et al., *First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: preliminary maps and basic results*, *Astrophys. J. Suppl.* **148** (2003) 1 [astro-ph/0302207];
G. Hinshaw et al., *First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: angular power spectrum*, *Astrophys. J. Suppl.* **148** (2003) 135 [astro-ph/0302217];
H.V. Peiris et al., *First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: implications for inflation*, *AstrophysJ̇. Suppl.* **148** (2003) 213 [astro-ph/0302225].

[9] N.D. Birrell and P.C.W. Davies, *Quantum fields in curved space*, Cambridge University Press, Cambridge 1982.

[10] F. Larsen and R. McNees, *Inflation and de Sitter holography*, *J. High Energy Phys.* **07** (2003) 051 [hep-th/0307026].

[11] C.P. Burgess, *Quantum gravity in everyday life: general relativity as an effective field theory*, *Living Rev. Rel.* **7** (2004) 5 [gr-qc/0311082].

[12] R.H. Brandenberger and J. Martin, *On signatures of short distance physics in the cosmic microwave background*, *Int. J. Mod. Phys.* **A 17** (2002) 3663 [hep-th/0202142].

[13] K. Symanzik, *Schrödinger representation and Casimir effect in renormalizable quantum field theory*, *Nucl. Phys.* **B 190** (1981) 1.

[14] P. Candelas, *Vacuum polarization in the presence of dielectric and conducting surfaces*, *Ann. Phys. (NY)* **143** (1982) 241.

[15] D. Deutsch and P. Candelas, *Boundary effects in quantum field theory*, *Phys. Rev.* **D 20** (1979) 3063.

[16] G. Barton, *Some surface effects in the hydrodynamic model of metals*, *Rept. Prog. Phys.***42** (1979) 963.

[17] E. Witten, *Multi-trace operators, boundary conditions and AdS/CFT correspondence*, hep-th/0112258.

[18] S. Fredenhagen, *Organizing boundary RG flows*, *Nucl. Phys.* **B 660** (2003) 436 [hep-th/0301229].

[19] O. Aharony, O. DeWolfe, D.Z. Freedman and A. Karch, *Defect conformal field theory and locally localized gravity*, *J. High Energy Phys.* **07** (2003) 030 [hep-th/0303249].

[20] Y. Aghababaie and C.P. Burgess, *Effective actions, boundaries and precision calculations of Casimir energies*, hep-th/0304066.

[21] K. Graham and G.M.T. Watts, *Defect lines and boundary flows*, *J. High Energy Phys.* **04** (2004) 019 [hep-th/0306167].

[22] R.L. Jaffe, *Unnatural acts: unphysical consequences of imposing boundary conditions on quantum fields*, *AIP Conf. Proc.* **687** (2003) 3 [hep-th/0307014].

[23] R.G. Leigh, *Dirac-Born-Infeld action from Dirichlet sigma model*, *Mod. Phys. Lett.* **A 4** (1989) 2767.

[24] J. Polchinski, *Renormalization and effective lagrangians*, *Nucl. Phys.* **B 231** (1984) 269.

[25] A.J. Hamilton, D. Kabat and M.K. Parikh, *Cosmological particle production without Bogolubov coefficients*, hep-th/0311180.

[26] D.J.H. Chung, A. Notari and A. Riotto, *Minimal theoretical uncertainties in inflationary predictions*, *JCAP* **10** (2003) 012 [hep-ph/0305074].

[27] A. Strominger, *The dS/CFT correspondence*, *J. High Energy Phys.* **10** (2001) 034 [hep-th/0106113].

[28] V. Balasubramanian, J. de Boer and D. Minic, *Mass, entropy and holography in asymptotically de Sitter spaces*, *Phys. Rev.* **D 65** (2002) 123508 [hep-th/0110108].

[29] F. Larsen, J.P. van der Schaar and R.G. Leigh, *de Sitter holography and the cosmic microwave background*, *J. High Energy Phys.* **04** (2002) 047 [hep-th/0202127].

[30] J.P. van der Schaar, *Inflationary perturbations from deformed CFT*, *J. High Energy Phys.* **01** (2004) 070 [hep-th/0307271].

[31] T. Banks and L. Mannelli, *de Sitter vacua, renormalization and locality*, *Phys. Rev.* **D 67** (2003) 065009 [hep-th/0209113];
M.B. Einhorn and F. Larsen, *Interacting quantum field theory in de Sitter vacua*, *Phys. Rev.* **D 67** (2003) 024001 [hep-th/0209159];
K. Goldstein and D.A. Lowe, *A note on alpha-vacua and interacting field theory in de Sitter space*, *Nucl. Phys.* **B 669** (2003) 325 [hep-th/0302050];
M.B. Einhorn and F. Larsen, *Squeezed states in the de Sitter vacuum*, *Phys. Rev.* **D 68** (2003) 064002 [hep-th/0305056];
H. Collins, R. Holman and M.R. Martin, *The fate of the alpha-vacuum*, *Phys. Rev.* **D 68** (2003) 124012 [hep-th/0306028];
K. Goldstein and D.A. Lowe, *Real-time perturbation theory in de Sitter space*, *Phys. Rev.* **D 69** (2004) 023507 [hep-th/0308135];
H. Collins and M.R. Martin, *The enhancement of inflaton loops in an alpha-vacuum*, hep-ph/0309265.

[32] H. Collins and R. Holman, *Taming the alpha vacuum*, hep-th/0312143.

[33] S.R. Coleman, J. Wess and B. Zumino, *Structure Of Phenomenological lagrangians, 1*, *Phys. Rev.* **177** (1969) 2239;
C.G. Callan, S.R. Coleman, J. Wess and B. Zumino, *Structure Of Phenomenological lagrangians, 2*, *Phys. Rev.* **177** (1969) 2247.

[34] H. Georgi, *On-shell effective field theory*, *Nucl. Phys.* **B 361** (1991) 339;
C. Arzt, *Reduced effective lagrangians*, *Phys. Lett.* **B 342** (1995) 189 [hep-ph/9304230];
C. Grosse-Knetter, *Effective lagrangians with higher derivatives and equations of motion*, *Phys. Rev.* **D 49** (1994) 6709 [hep-ph/9306321].

JHEP04(2004)076